Contents

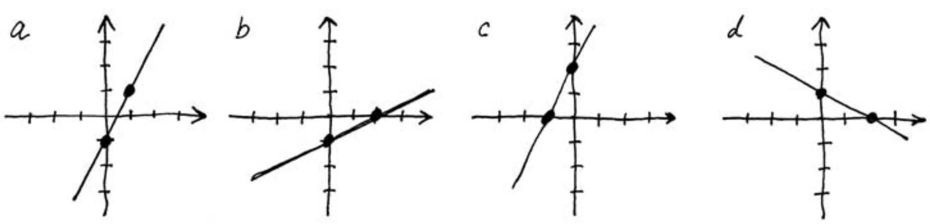
DIAGNOSTIC TEST

Are you ready to study calculus?

Algebra is the language in which we express the ideas of calculus. Therefore, to understand calculus and express its ideas with precision, you need to know some algebra.

If you are comfortable with the algebra covered in the following problems, you are ready to begin your study of calculus. If not, turn to the *Algebra Appendix* beginning on page A.xxx and review the *Complete Solutions* to these problems, and continue reading the other parts of the Appendix that cover anything that you do not know.

Problems	**Answers**
1. True or False? $\frac{1}{2} < -3$	False
2. Express $\{x \mid -4 < x \leq 5\}$ in interval notation.	$(-4, 5]$
3. What is the slope of the line through the points $(6, -7)$ and $(9, 8)$?	5
4. On the line $y = 3x + 4$, what value of $\Delta\, y$ corresponds to$\Delta\ x = 2$?	6
5. Which sketch shows the graph of the line $y = 2x - 1$?	a



6. True or False? $\left(\frac{\sqrt{x}}{y}\right)^{-2} = \frac{y^2}{x}$	True
7. Find the zeros of the function $f(x) = 9x^2 - 6x - 1$	$x = \frac{1 \pm \sqrt{2}}{3}$
8. Expand and simplify $x(8 - x) - (3x + 7)$.	$-x^2 + 5x - 7$
9. What is the domain of $f(x) = \dfrac{x^2 - 3x + 2}{x^3 + x^2 - 6x}$?	$\{x \mid x \neq -3, x \neq 0, x \neq 2\}$
10. For $f(x) = x^2 - 5x$, find the difference quotient $\dfrac{f(x+h) - f(x)}{h}$.	$2x - 5 + h$

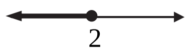
Chapter 1: Functions

EXERCISES 1.1

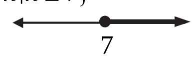
1. $\{x \mid 0 \le x < 6\}$

2. $\{x \mid -3 < x \le 5\}$

3. $\{x \mid x \le 2\}$



4. $\{x \mid x \ge 7\}$



5. **a.** Since $\Delta x = 3$ and $m = 5$, then Δy, the change in y, is

$$\Delta y = 3 \cdot m = 3 \cdot 5 = 15$$

b. Since $\Delta x = -2$ and $m = 5$, then Δy, the change in y, is

$$\Delta y = -2 \cdot m = -2 \cdot 5 = -10$$

6. **a.** Since $\Delta x = 5$ and $m = -2$, then Δy, the change in y, is

$$\Delta y = 5 \cdot m = 5 \cdot (-2) = -10$$

b. Since $\Delta x = -4$ and $m = -2$, then Δy, the change in y, is

$$\Delta y = -4 \cdot m = -4 \cdot (-2) = 8$$

7. For (2, 3) and (4, –1), the slope is

$$\frac{-1-3}{4-2} = \frac{-4}{2} = -2$$

8. For (3, –1) and (5, 7), the slope is

$$\frac{7-(-1)}{5-3} = \frac{7+1}{2} = \frac{8}{2} = 4$$

9. For (–4, 0) and (2, 2), the slope is

$$\frac{2-0}{2-(-4)} = \frac{2}{2+4} = \frac{2}{6} = \frac{1}{3}$$

10. For (–1, 4) and (5, 1), the slope is

$$\frac{1-4}{5-(-1)} = \frac{-3}{5+1} = \frac{-3}{6} = -\frac{1}{2}$$

11. For (0, –1) and (4, –1), the slope is

$$\frac{-1-(-1)}{4-0} = \frac{-1+1}{4} = \frac{0}{4} = 0$$

12. For $\left(-2, \frac{1}{2}\right)$ and $\left(5, \frac{1}{2}\right)$, the slope is

$$\frac{\frac{1}{2}-\frac{1}{2}}{5-(-2)} = \frac{0}{5+2} = \frac{0}{7} = 0$$

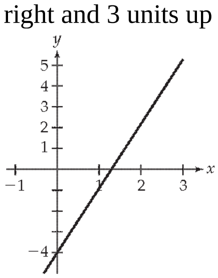
13. For (2, –1) and (2, 5), the slope is

$$\frac{5-(-1)}{2-2} = \frac{5+1}{0} \qquad \text{undefined}$$

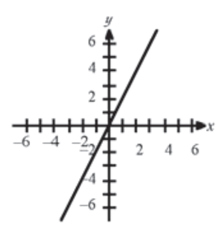
14. For (6, –4) and (6, –3), the slope is

$$\frac{-3-(-4)}{6-6} = \frac{-3+4}{0} \qquad \text{undefined}$$

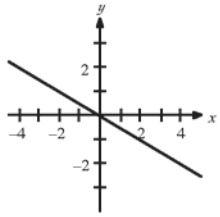
15. Since $y = 3x - 4$ is in slope-intercept form, $m = 3$ and the y-intercept is (0, –4). Using the slope $m = 3$, we see that the point 1 unit to the right and 3 units up is also on the line.



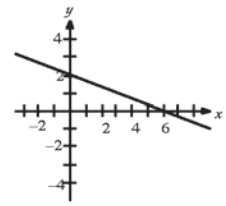
16. Since $y = 2x$ is in slope-intercept form, $m = 2$ and the y-intercept is (0, 0). Using $m = 2$, we see that the point 1 to the right and 2 units up is also on the line.



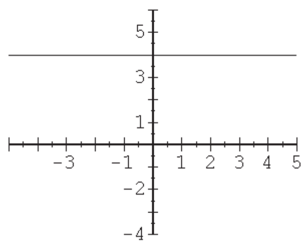
17. Since $y = -\frac{1}{2}x$ is in slope-intercept form, $m = -\frac{1}{2}$ and the y-intercept is (0, 0). Using $m = -\frac{1}{2}$, we see that the point 2 units to the right and 1 unit down is also on the line.



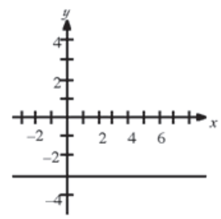
18. Since $y = -\frac{1}{3}x + 2$ is in slope-intercept form, $m = -\frac{1}{3}$ and the y-intercept is (0, 2). Using $m = -\frac{1}{3}$, we see that the point 3 units to the right and 1 unit down is also on the line.



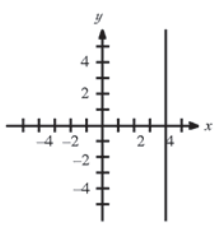
19. The equation $y = 4$ is the equation of the horizontal line through all points with y-coordinate 4. Thus, $m = 0$ and the y-intercept is (0, 4).



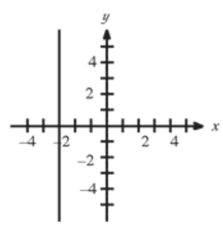
20. The equation $y = –3$ is the equation of the horizontal line through all points with y-coordinate –3. Thus, $m = 0$ and the y-intercept is (0, –3).



21. The equation $x = 4$ is the equation of the vertical line through all points with x-coordinate 4. Thus, m is not defined and there is no y-intercept.



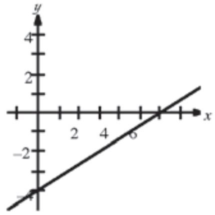
22. The equation $x = –3$ is the equation of the vertical line through all points with x-coordinate –3. Thus, m is not defined and there is no y-intercept.



23. First, solve for y:

$$\begin{aligned} 2x - 3y &= 12 \\ -3y &= -2x + 12 \\ y &= \frac{2}{3}x - 4 \end{aligned}$$

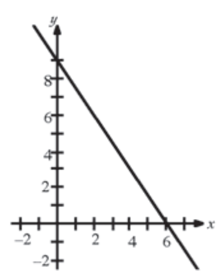
Therefore, $m = \frac{2}{3}$ and the y-intercept is (0, –4).



24. First, solve for y:

$$\begin{aligned} 3x + 2y &= 18 \\ 2y &= -3x + 18 \\ y &= -\frac{3}{2}x + 9 \end{aligned}$$

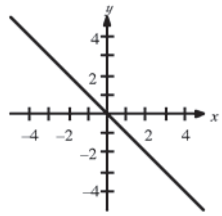
Therefore, $m = -\frac{3}{2}$ and the y-intercept is (0, 9).



25. First, solve for y:

$$x + y = 0$$
$$y = -x$$

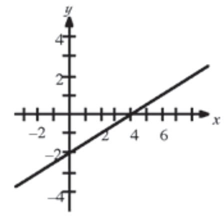
Therefore, $m = -1$ and the y-intercept is (0, 0).



26. First, solve for y:

$$x = 2y + 4$$
$$-2y = -x + 4$$
$$y = \frac{1}{2}x - 2$$

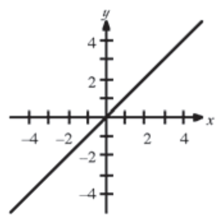
Therefore, $m = \frac{1}{2}$ and the y-intercept is (0, –2).



27. First, solve for y:

$$x - y = 0$$
$$-y = -x$$
$$y = x$$

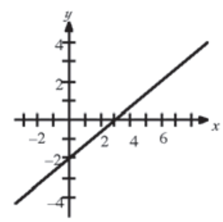
Therefore, $m = 1$ and the y-intercept is (0, 0).



28. First, put the equation in slope-intercept form:

$$y = \frac{2}{3}(x - 3)$$
$$y = \frac{2}{3}x - 2$$

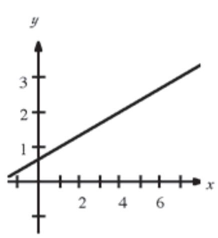
Therefore, $m = \frac{2}{3}$ and the y-intercept is (0, –2).



29. First, put the equation in slope-intercept form:

$$y = \frac{x+2}{3}$$
$$y = \frac{1}{3}x + \frac{2}{3}$$

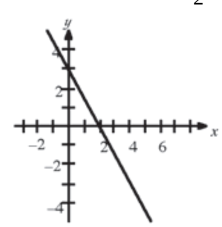
Therefore, $m = \frac{1}{3}$ and the y-intercept is $\left(0, \frac{2}{3}\right)$.



30. First, solve for y:

$$\frac{x}{2} + \frac{y}{3} = 1$$
$$\frac{y}{3} = -\frac{1}{2}x + 1$$
$$y = -\frac{3}{2}x + 3$$

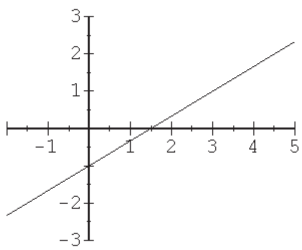
Therefore, $m = -\frac{3}{2}$ and the y-intercept is (0, 3).



31. First, solve for y:

$$\frac{2x}{3} - y = 1$$
$$-y = -\frac{2}{3}x + 1$$
$$y = \frac{2}{3}x - 1$$

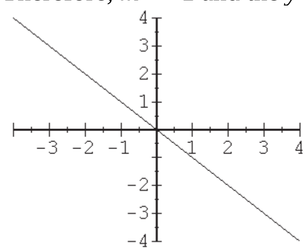
Therefore, $m = \frac{2}{3}$ and the y-intercept is (0, –1).



32. First, solve for y:

$$\frac{x+1}{2} + \frac{y+1}{2} = 1$$
$$x + 1 + y + 1 = 2$$
$$x + y + 2 = 2$$
$$y = -x$$

Therefore, m = –1 and the y-intercept is (0, 0).



33. $y = -2.25x + 3$

34. $y = \frac{2}{3}x - 8$

35.
$$y - (-2) = 5[x - (-1)]$$
$$y + 2 = 5x + 5$$
$$y = 5x + 3$$

36.
$$y - 3 = -1(x - 4)$$
$$y - 3 = -x + 4$$
$$y = -x + 7$$

37. $y = -4$

38. $y = \frac{3}{4}$

39. $x = 1.5$

40. $x = \frac{1}{2}$

41. First, find the slope.

$$m = \frac{-1-3}{7-5} = \frac{-4}{2} = -2$$

Then use the point-slope formula with this slope and the point (5, 3).

$$y - 3 = -2(x - 5)$$
$$y - 3 = -2x + 10$$
$$y = -2x + 13$$

42. First, find the slope.

$$m = \frac{0-(-1)}{6-3} = \frac{1}{3}$$

Then use the point-slope formula with this slope and the point (6, 0).

$$y - 0 = \frac{1}{3}(x - 6)$$
$$y = \frac{1}{3}x - 2$$

43. First, find the slope.

$$m = \frac{-1-(-1)}{5-1} = \frac{-1+1}{4} = 0$$

Then use the point-slope formula with this slope and the point (1,–1).

$$y - (-1) = 0(x - 1)$$
$$y + 1 = 0$$
$$y = -1$$

44. First, find the slope.

$$m = \frac{-4-0}{2-2} = \frac{-4}{0} \qquad \text{undefined}$$

Since the slope of the line is undefined, the line is a vertical line. Because the x-coordinates of the points are 2, the equation is $x = 2$.

45. **a.** First find the slope of the line $4y-3x=5$. Write the equation in slope-intercept form.

$$y=\frac{3}{4}x+\frac{5}{4}$$

The slope of the parallel line is $m=\frac{3}{4}$.

Next, use the point-slope form with the point (12, 2):

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-2&=\frac{3}{4}(x-12)\\ y&=\frac{3}{4}x-7 \end{aligned}$$

b. The slope of the line perpendicular to $y=\frac{3}{4}x+\frac{5}{4}$. is $m=-\frac{4}{3}$.

Next, use the point-slope form with the point (12, 2):

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-2&=-\frac{4}{3}(x-12)\\ y&=-\frac{4}{3}x+18 \end{aligned}$$

46. **a.** First find the slope of the line $x+3y=7$. Write the equation in slope-intercept form.

$$y=-\frac{1}{3}x+\frac{7}{3}.$$

The slope of the parallel line is $m=-\frac{1}{3}$.

Next, use the point-slope form with the point (–6, 5):

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-5&=-\frac{1}{3}(x+6)\\ y&=-\frac{1}{3}x+3 \end{aligned}$$

b. The slope of the line perpendicular to $y=-\frac{1}{3}x+\frac{7}{3}$. is $m=3$.

Next, use the point-slope form with the point (–6, 5):

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-5&=3(x+6)\\ y&=3x+23 \end{aligned}$$

47. The y-intercept of the line is (0, 1), and $\Delta y = -2$ for $\Delta x = 1$. Thus, $m=\frac{\Delta y}{\Delta x}=\frac{-2}{1}=-2$. Now, use the slope-intercept form of the line: $y=-2x+1$.

48. The y-intercept of the line is (0, –2), and $\Delta y = 3$ for $\Delta x = 1$. Thus, $m=\frac{\Delta y}{\Delta x}=\frac{3}{1}=3$. Now, use the slope-intercept form of the line: $y=3x-2$

49. The y-intercept is (0, –2), and $\Delta y = 3$ for $\Delta x = 2$. Thus, $m=\frac{\Delta y}{\Delta x}=\frac{3}{2}$. Now, use the slope-intercept form of the line: $y=\frac{3}{2}x-2$

50. The y-intercept is (0, 1), and $\Delta y = -2$ for $\Delta x = 3$. Thus, $m=\frac{\Delta y}{\Delta x}=\frac{-2}{3}=-\frac{2}{3}$. Now, use the slope-intercept form of the line: $y=-\frac{2}{3}x+1$

51. First, consider the line through the points (0, 5) and (5, 0). The slope of this line is $m=\frac{0-5}{5-0}=\frac{-5}{5}=-1$. Since (0, 5) is the y-intercept of this line, use the slope-intercept form of the line: $y=-1x+5$ or $y=-x+5$.
Now consider the line through the points (5, 0) and (0, –5). The slope of this line is $m=\frac{-5-0}{0-5}=\frac{-5}{-5}=1$. Since (0,–5) is the y-intercept of the line, use the slope-intercept form of the line: $y=1x-5$ or $y=x-5$
Next, consider the line through the points (0, –5) and (–5, 0). The slope of this line is $m=\frac{0-(-5)}{-5-0}=\frac{5}{-5}=-1$. Since (0, –5) is the y-intercept, use the slope-intercept form of the line: $y=-1x-5$ or $y=-x-5$
Finally, consider the line through the points (–5, 0) and (0, 5). The slope of this line is $m=\frac{5-0}{0-(-5)}=\frac{5}{5}=1$. Since (0, 5) is the y-intercept, use the slope-intercept form of the line: $y=1x+5$ or $y=x+5$

52. The equation of the vertical line through (5, 0) is $x = 5$.
The equation of the vertical line through (–5, 0) is $x = -5$.
The equation of the horizontal line through (0, 5) is $y = 5$.
The equation of the horizontal line through (0, –5) is $y = -5$.

53. If the point (x_1, y_1) is the y-intercept $(0, b)$, then substituting into the point-slope form of the line gives

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-b&=m(x-0)\\ y-b&=mx\\ y&=mx+b \end{aligned}$$

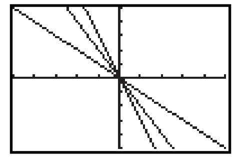
54. To find the x-intercept, substitute $y = 0$ into the equation and solve for x:

$$\frac{x}{a}+\frac{y}{b}=1$$
$$\frac{x}{a}+\frac{0}{b}=1$$
$$\frac{x}{a}=1$$
$$x=a$$ Thus, $(a, 0)$ is the x-intercept.

To find the y-intercept, substitute $x = 0$ into the equation and solve for y:

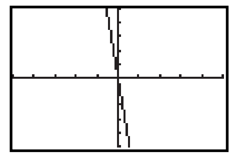
$$\frac{x}{a}+\frac{y}{b}=1$$
$$\frac{0}{a}+\frac{y}{b}=1$$
$$\frac{y}{b}=1$$
$$y=b$$ Thus, $(0, b)$ is the y-intercept.

55. **a.**



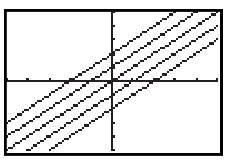
on [–5, 5] by [–5, 5]

b.



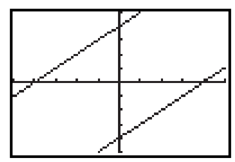
on [–5, 5] by [–5, 5]

56. **a.**



on [–5, 5] by [–5, 5]

b.



on [–5, 5] by [–5, 5]

57. Low demand: [0, 8);
average demand: [8, 20);
high demand: [20, 40);
critical demand: $[40, \infty)$

58. A: [90, 100]; B: [80,90); C: [70, 80);
D: [60, 70); F: [0, 60)

59. **a.** The value of x corresponding to the year 2020 is $x = 2020 - 1900 = 120$. Substituting $x = 120$ into the equation for the regression line gives

$$y=-0.356x+257.44$$
$$y=-0.356(120)+257.44=214.72 \text{ seconds}$$

Since 3 minutes = 180 seconds, 214.72 = 3 minutes 34.72 seconds. Thus, the world record in the year 2020 will be 3 minutes 34.72 seconds.

b. To find the year when the record will be 3 minutes 30 seconds, first convert 3 minutes 30 seconds to seconds: 3 minutes 30 seconds = 3 minutes • $\dfrac{60 \text{ sec}}{1 \text{ min}}$ + 30 seconds = 210 seconds.

Now substitute $y = 210$ seconds into the equation for the regression line and solve for x.

$$\begin{aligned} y &= -0.356x+257.44 \\ 210 &= -0.356x+257.44 \\ 0.356x &= 257.44-210 \\ 0.356x &= 47.44 \\ x &= \frac{47.44}{0.356} \approx 133.26 \end{aligned}$$

Since x represents the number of years after 1900, the year corresponding to this value of x is $1900 + 133.26 = 2033.26 \approx 2033$. The world record will be 3 minutes 30 seconds in 2033.

60. For $x = 720$:

$$\begin{aligned} y &= -0.356x+257.44 \\ &= -0.356(720)+257.44 \\ &= -256.32+257.44 = 1.12 \text{ seconds} \end{aligned}$$

For $x = 722$:

$$\begin{aligned} y &= -0.356x+257.44 \\ &= -0.356(722)+257.44 \\ &= -257.744+257.44 = 0.408 \text{ second} \end{aligned}$$

These are both unreasonable times for running 1 mile.

61. **a.** To find the linear equation, first find the slope of the line containing these points.

$$m = \frac{146-70}{3-1} = \frac{76}{2} = 38$$

Next, use the point-slope form with the point (1, 70):

$$y - y_1 = m(x - x_1)$$
$$y - 70 = 38(x-1)$$
$$y = 38x + 32$$

b. Sales are increasing by 38 million units per year.

c. The sales at the end of 2020 is y = 38(10) + 32 = 412 million units.

62. **a.** First, find the slope of the line containing the points.

$$m = \frac{42.8-38.6}{4-1} = \frac{4.2}{3} = 1.4$$

Next, use the point-slope form with the point (1, 38.6):

$$y - y_1 = m(x - x_1)$$
$$y - 38.6 = 1.4(x-1)$$
$$y = 1.4x + 37.2$$

b. PCPI increases by about \$1400 (or \$1.4 thousand) each year.

c. The value of x corresponding to 2020 is x = 2020 – 2008 = 12. Substitute 12 into the equation:
y = 1.4(12) + 37.2 = \$54 thousand or \$54,000

63. **a.** First, find the slope of the line containing the points.

$$m = \frac{212-32}{100-0} = \frac{180}{100} = \frac{9}{5}$$

Next, use the point-slope form with the point (0, 32):

$$y - y_1 = m(x - x_1)$$
$$y - 32 = \frac{9}{5}(x-0)$$
$$y = \frac{9}{5}x + 32$$

b. Substitute 20 into the equation.

$$y = \frac{9}{5}x + 32$$
$$y = \frac{9}{5}(20) + 32 = 36 + 32 = 68^\circ \text{F}$$

64. **a.** First, find the slope of the line containing the points.

$$m = \frac{89.8-74.8}{4-0} = \frac{15}{4} = 3.75$$

Next, use the point-slope form with the point (0, 74.8):

$$y - y_1 = m(x - x_1)$$
$$y - 74.8 = 3.75(x-0)$$
$$y = 3.75x + 74.8$$

b. Since 2021 is 12 years after 2009, substitute 11 into the equation.

$$y = 3.75x + 74.8$$
$$y = 3.75(12) + 74.8 = 119.8 \text{ thousand dollars}$$

or \$119,800

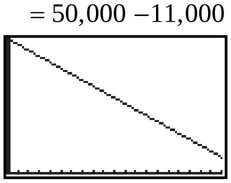
65. **a.** Price = \$50,000; useful lifetime = 20 years; scrap value = \$6000

$$V = 50{,}000 - \left(\frac{50{,}000 - 6000}{20}\right)t \quad 0 \le t \le 20$$
$$= 50{,}000 - 2200t \quad 0 \le t \le 20$$

b. Substitute t = 5 into the equation.

$$V = 50{,}000 - 2200t$$
$$= 50{,}000 - 2200(5)$$
$$= 50{,}000 - 11{,}000 = \$39{,}000$$

c.



on [0, 20] by [0, 50,000]

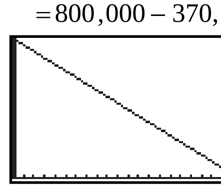
66. **a.** Price = \$800,000; useful lifetime = 20 yrs; scrap value = \$60,000

$$V = 800{,}000 - \left(\frac{800{,}000 - 60{,}000}{20}\right)t \quad 0 \le t \le 20$$
$$= 800{,}000 - 37{,}000t \quad 0 \le t \le 20$$

b. Substitute t = 10 into the equation.

$$V = 800{,}000 - 37{,}000t$$
$$= 800{,}000 - 37{,}000(10)$$
$$= 800{,}000 - 370{,}000 = \$430{,}000$$

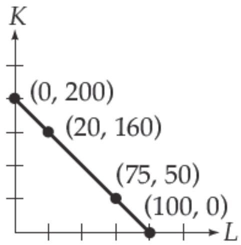
c.



on [0, 20] by [0, 800,000]

67. **a.** Substitute $w = 10$, $r = 5$, $C = 1000$ into the equation.

$10L + 5K = 1000$



b. Substitute each pair into the equation.

For (100, 0), $10(100) + 5(0) = 1000$

For (75, 50), $10(75) + 5(50) = 1000$

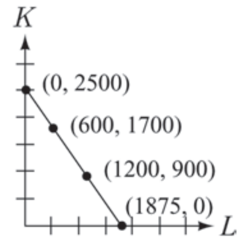
For (20, 160), $10(20) + 5(160) = 1000$

For (0, 200), $10(0) + 5(200) = 1000$

Every pair gives 1000.

68. **a.** Substitute $w = 8$, $r = 6$, $C = 15{,}000$ into the equation.

$8L + 6K = 15{,}000$



b. Substitute each pair into the equation.

For (1875, 0), $8(1875) + 6(0) = 15{,}000$

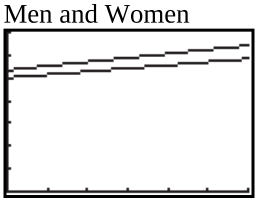
For (1200, 900), $8(1200) + 6(900) = 15{,}000$

For (600, 1700), $8(600) + 6(1700) = 15{,}000$

For (0, 2500), $8(0) + 6(2500) = 15{,}000$

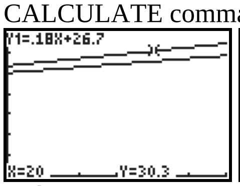
Every pair gives 15,000.

69. **a.** Median Marriage Age for Men and Women

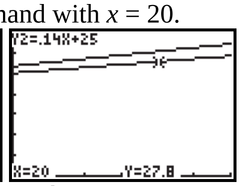


on [0, 30] by [0, 35]

b. The x-value corresponding to the year 2020 is $x = 2020 - 2000 = 20$. The following screens are a result of the CALCULATE command with $x = 20$.



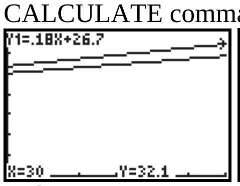
Median Age at Marriage for Men in 2020



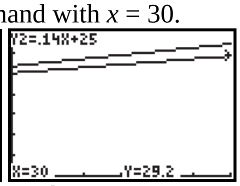
Median Age at Marriage for Women in 2020.

So, the median marriage age for men in 2020 will be 30.3 years and for women it will be 27.8 years.

c. The x-value corresponding to the year 2030 is $x = 2030 - 2000 = 30$. The following screens are a result of the CALCULATE command with $x = 30$.



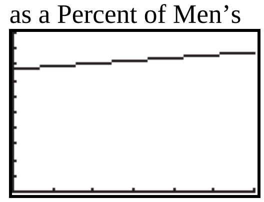
Median Age at Marriage for Men in 2030



Median Age at Marriage for Women in 2030.

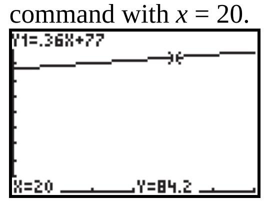
So, the median marriage age for men in 2030 will be 32.1 years and for women it will be 29.2 years.

70. **a.** Women's Annual Earnings as a Percent of Men's



on [0, 30] by [0, 100]

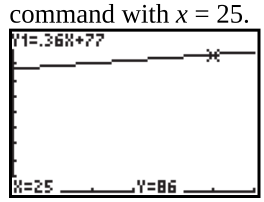
b. The x-value corresponding to the year 2020 is $x = 2020 - 2000 = 20$. The following screen is a result of the CALCULATE command with $x = 20$.



Women's Annual Earnings as a Percent of Men's

So, in the year 2020 women's wages will be about 84.2% of men's wages.

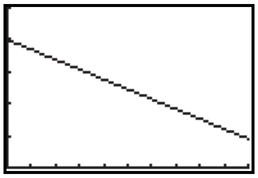
c. The x-value corresponding to the year 2025 is $x = 2025 - 2000 = 25$. The following screen is a result of the CALCULATE command with $x = 25$.



Women's Annual Earnings as a Percent of Men's

So in the year 2025 women's wages will be about 86% of men's wages.

71. **a.**



on [0, 100] by [0, 50]

b. To find the probability that a person with a family income of $40,000 is a smoker, substitute 40 into the equation
$y = -0.31x + 40$
$y = -0.31(40) + 40 = 27.6$ or 28%.

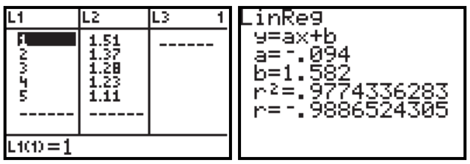
c. The probability that a person with a family income of $70,000 is a smoker is
$y = -0.31(70) + 40 = 18.3$ or 18%.

72. **a.** To find the reported "happiness" of a person with an income of $25,000, substitute 25 into the equation
$y = 0.065x - 0.613$
$y = 0.065(25) - 0.613 = 1.0.$

b. The reported "happiness" of a person with an income of $35,000 is
$y = 0.065(35) - 0.613 = 1.7.$

c. The reported "happiness" of a person with an income of $45,000 is
$y = 0.065(45) - 0.613 = 2.3.$

73. **a.**

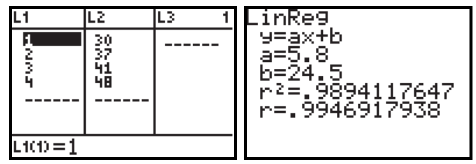


$y = -0.094x + 1.582$

b. Cigarette consumption is declining by about 94 cigarettes (from 0.094 thousand, so about 5 packs) per person per year.

c. $y = -0.094(13) + 1.582 = 0.36$ thousand (360 cigarettes)

74. **a.**

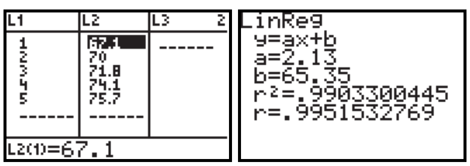


$y = 5.8x + 24.5$

b. Each year the usage increases by about 5.8 percentage points.

c. $y = 5.8(11) + 24.5 = 88.3\%$

75. **a.**

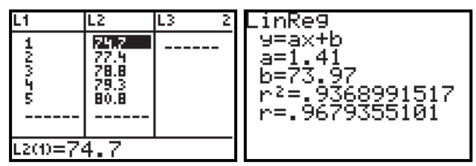


$y = 2.13x + 65.35$

b. The male life expectancy is increasing by 2.13 years per decade, which is 0.213 years (or about 2.6 months per year).

c. $y = 2.13(6.5) + 65.35 \approx 79.2$ years

76. **a.**

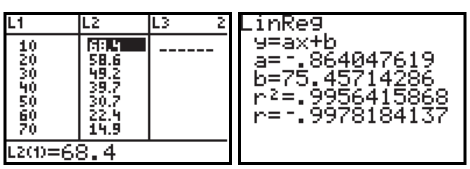


$y = 1.41x + 73.97$

b. The female life expectancy is increasing by 1.41 years per decade, which is 0.141 years (or about 1.7 months per year).

c. $y = 1.41(6.5) + 73.97 \approx 83.1$ years

77. **a.**



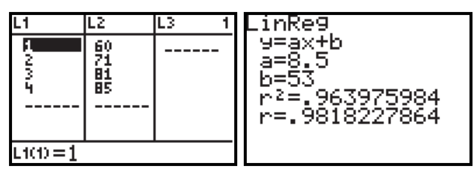
$y = -0.864x + 75.46$

b. Future longevity decreases by 0.864 (or about 10.44 months) per year.

c. $y = -0.864(25) + 75.46 \approx 53.9$ years

d. It would not make sense to use the regression line to predict future longevity at age 90 because the line predicts –2.3 years of life remaining.

78. **a.**



$y = 8.5x + 53$

b. Seat belt use increases by 8.5% each 5 years (or about 1.7% per year).

c. $y = 8.5(5.4) + 53 = 98.9\%$

d. It would not make sense to use the regression line to predict seat belt use in 2025 ($x = 7$) because the line predicts 112.5%.

79. False: Infinity is not a number.

80. True: All negative numbers must be less than zero, and all positive numbers are more than zero. Therefore, all negative numbers are less than all positive numbers.

81. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ for any two points (x_1, y_1) and (x_1, y_1) on the line *or* the slope is the amount that the line rises when *x* increases by 1.

82. "Slope" is the answer to the first blank. The second blank would be describing it as negative, because the slope of a line slanting downward as you go to the right is a "fall" over "run".

83. False: The slope of a vertical line is undefined.

84. False: The slope of a vertical line is undefined, so a vertical line does not have a slope.

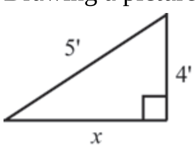
85. True: The slope is $\dfrac{-a}{b}$ and the *y*-intercept is $\dfrac{c}{b}$.

86. True: $x = c$ will always be a vertical line because the *x* values do not change.

87. False: It should be $\dfrac{y_2 - y_1}{x_2 - x_1}$.

88. False. A vertical line has no slope, so there is no *m* for $y = mx + b$.

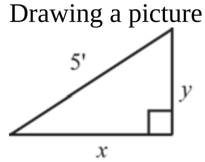
89. Drawing a picture of a right triangle.



$$\begin{aligned} x^2 + 4^2 &= 5^2 \\ x^2 + 16 &= 25 \\ x^2 &= 9 \\ x &= 3 \end{aligned}$$

The slope is $m = \dfrac{4}{3}$ or $-\dfrac{4}{3}$ if the ladder slopes downward.

90. Drawing a picture of a right triangle.



$$x^2 + y^2 = 5^2$$

$$\frac{y}{x} = 0.75 \Rightarrow y = 0.75x$$

$$\begin{aligned} x^2 + (0.75x)^2 &= 5^2 \\ x^2 + 0.5625x^2 &= 25 \\ 1.5625x^2 &= 25 \\ x^2 &= 16 \\ x &= 4 \\ y &= 0.75(4) = 3 \end{aligned}$$

The upper end is 3 feet high.

91. To find the *x*-intercept, substitute $y = 0$ into the equation and solve for *x:*

$$\begin{aligned} y &= mx + b \\ 0 &= mx + b \\ -mx &= b \\ x &= -\frac{b}{m} \end{aligned}$$

If $m \neq 0$, then a single *x*-intercept exists. So $a = -\dfrac{b}{m}$. Thus, the *x*-intercept is $\left(-\dfrac{b}{m}, 0\right)$.

92. **i.** To obtain the slope-intercept form of a line, solve the equation for *y*:

$$\begin{aligned} ax + by &= c \\ by &= -ax + c \\ y &= -\frac{a}{b}x + \frac{c}{b} \end{aligned}$$

ii. Substitute 0 for *b* and solve for *x:*

$$\begin{aligned} ax + by &= c \\ ax + 0 \cdot y &= c \\ ax &= c \\ x &= \frac{c}{a} \end{aligned}$$

93. Consider $R > 1$ and $0 < x < K$

$x < K$ means that $K - x > 0$ and $0 < \frac{x}{K} < 1$.

Since $K - x > 0$, then

$$K - x + Rx > Rx$$
$$K + (R-1)x > Rx$$
$$K\left[1 + \left(\frac{R-1}{K}\right)x\right] > Rx$$

Therefore, $K > \dfrac{Rx}{1 + \left(\frac{R-1}{K}\right)x} = y$

Additionally, since $0 < \frac{x}{K} < 1$,

$$1 + (R-1)\left(\frac{x}{K}\right) < 1 + (R-1)\cdot 1$$

So, $y = \dfrac{Rx}{1 + \left(\frac{R-1}{K}\right)x} > \dfrac{Rx}{1 + (R-1)} = \dfrac{Rx}{R} = x$

We have $x < y < K$.

94. $x > K$ means that $K - x < 0$ and $\frac{x}{K} > 1$.

Since $K - x < 0$, then

$$K - x + Rx < Rx$$
$$K + (R-1)x < Rx$$
$$K\left[1 + \left(\frac{R-1}{K}\right)x\right] < Rx$$

Therefore, $K < \dfrac{Rx}{1 + \left(\frac{R-1}{K}\right)x} = y$

Additionally, since $\frac{x}{K} > 1$,

$$1 + (R-1)\left(\frac{x}{K}\right) > 1 + (R-1)\cdot 1$$

So, $y = \dfrac{Rx}{1 + \left(\frac{R-1}{K}\right)x} < \dfrac{Rx}{1 + (R-1)} = \dfrac{Rx}{R} = x$

We have $K < y < x$.

EXERCISES 1.2

1. $\left(2^2 \cdot 2\right)^2 = \left(2^2 \cdot 2^1\right)^2 = \left(2^3\right)^2 = 2^6 = 64$

2. $\left(5^2 \cdot 4\right)^2 = \left(5^2 \cdot 2^2\right)^2 = \left(10^2\right)^2 = 10^4 = 10{,}000$

3. $2^{-4} = \dfrac{1}{2^4} = \dfrac{1}{16}$

4. $3^{-3} = \dfrac{1}{3^3} = \dfrac{1}{27}$

5. $\left(\frac{1}{2}\right)^{-3} = \left(2^{-1}\right)^{-3} = 2^3 = 8$

6. $\left(\frac{1}{3}\right)^{-2} = \left(3^{-1}\right)^{-2} = 3^2 = 9$

7. $\left(\frac{5}{8}\right)^{-1} = \frac{8}{5}$

8. $\left(\frac{3}{4}\right)^{-1} = \frac{4}{3}$

9. $4^{-2} \cdot 2^{-1} = \left(2^2\right)^{-2} \cdot 2^{-1}$
$= 2^{-4} \cdot 2^{-1} = 2^{-5} = \dfrac{1}{2^5} = \dfrac{1}{32}$

10. $3^{-2} \cdot 9^{-1} = 3^{-2} \cdot \left(3^2\right)^{-1}$
$= 3^{-2} \cdot 3^{-2} = 3^{-4} = \dfrac{1}{3^4} = \dfrac{1}{81}$

11. $\left(\frac{3}{2}\right)^{-3} = \left(\frac{2}{3}\right)^3 = \dfrac{2^3}{3^3} = \dfrac{8}{27}$

12. $\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3 = \dfrac{3^3}{2^3} = \dfrac{27}{8}$

13. $\left(\frac{1}{3}\right)^{-2} - \left(\frac{1}{2}\right)^{-3} = \left(\frac{3}{1}\right)^2 - \left(\frac{2}{1}\right)^3 = \dfrac{3^2}{1^2} - \dfrac{2^3}{1^3}$
$= 9 - 8 = 1$

14. $\left(\frac{1}{3}\right)^{-2} - \left(\frac{1}{2}\right)^{-2} = \left(\frac{3}{1}\right)^2 - \left(\frac{2}{1}\right)^2 = \dfrac{3^2}{1^2} - \dfrac{2^2}{1^2}$
$= 9 - 4 = 5$

15. $\left[\left(\frac{2}{3}\right)^{-2}\right]^{-1} = \left[\left(\frac{3}{2}\right)^2\right]^{-1} = \left(\frac{3}{2}\right)^{-2} = \left(\frac{2}{3}\right)^2 = \dfrac{2^2}{3^2} = \dfrac{4}{9}$

16. $\left[\left(\frac{2}{5}\right)^{-2}\right]^{-1} = \left[\left(\frac{5}{2}\right)^2\right]^{-1} = \left(\frac{5}{2}\right)^{-2} = \left(\frac{2}{5}\right)^2 = \dfrac{2^2}{5^2} = \dfrac{4}{25}$

17. $25^{1/2} = \sqrt{25} = 5$

18. $36^{1/2} = \sqrt{36} = 6$

19. $25^{3/2} = \left(\sqrt{25}\right)^3 = 5^3 = 125$

20. $16^{3/2} = \left(\sqrt{16}\right)^3 = 4^3 = 64$

21. $16^{3/4} = \left(\sqrt[4]{64}\right)^3 = 2^3 = 8$

22. $27^{2/3} = \left(\sqrt[3]{27}\right)^2 = 3^2 = 9$

23. $(-8)^{2/3} = \left(\sqrt[3]{-8}\right)^2 = (-2)^2 = 4$

24. $(-27)^{2/3} = \left(\sqrt[3]{-27}\right)^2 = (-3)^2 = 9$

25. $(-8)^{5/3} = \left(\sqrt[3]{-8}\right)^5 = (-2)^5 = -32$

26. $(-27)^{5/3} = \left(\sqrt[3]{-27}\right)^5 = (-3)^5 = -243$

27. $\left(\frac{25}{36}\right)^{3/2}=\left(\sqrt{\frac{25}{36}}\right)^3=\left(\frac{5}{6}\right)^3=\frac{5^3}{6^3}=\frac{125}{216}$

28. $\left(\frac{16}{25}\right)^{3/2}=\left(\sqrt{\frac{16}{25}}\right)^3=\left(\frac{4}{5}\right)^3=\frac{4^3}{5^3}=\frac{64}{125}$

29. $\left(\frac{27}{125}\right)^{2/3}=\left(\sqrt[3]{\frac{27}{125}}\right)^2=\left(\frac{3}{5}\right)^2=\frac{9}{25}$

30. $\left(\frac{125}{8}\right)^{2/3}=\left(\sqrt[3]{\frac{125}{8}}\right)^2=\left(\frac{5}{2}\right)^2=\frac{25}{4}$

31. $\left(\frac{1}{32}\right)^{2/5}=\left(\sqrt[5]{\frac{1}{32}}\right)^2=\left(\frac{1}{2}\right)^2=\frac{1}{4}$

32. $\left(\frac{1}{32}\right)^{3/5}=\left(\sqrt[5]{\frac{1}{32}}\right)^3=\left(\frac{1}{2}\right)^3=\frac{1}{8}$

33. $4^{-1/2}=\frac{1}{4^{1/2}}=\frac{1}{\sqrt{4}}=\frac{1}{2}$

34. $9^{-1/2}=\frac{1}{9^{1/2}}=\frac{1}{\sqrt{9}}=\frac{1}{3}$

35. $4^{-3/2}=\frac{1}{4^{3/2}}=\frac{1}{\left(\sqrt{4}\right)^3}=\frac{1}{2^3}=\frac{1}{8}$

36. $9^{-3/2}=\frac{1}{9^{3/2}}=\frac{1}{\left(\sqrt{9}\right)^3}=\frac{1}{3^3}=\frac{1}{27}$

37. $8^{-2/3}=\frac{1}{8^{2/3}}=\frac{1}{\left(\sqrt[3]{8}\right)^2}=\frac{1}{2^2}=\frac{1}{4}$

38. $16^{-3/4}=\frac{1}{16^{3/4}}=\frac{1}{\left(\sqrt[4]{16}\right)^3}=\frac{1}{2^3}=\frac{1}{8}$

39. $(-8)^{-1/3}=\frac{1}{(-8)^{1/3}}=\frac{1}{\left(\sqrt[3]{-8}\right)}=\frac{1}{-2}=-\frac{1}{2}$

40. $(-27)^{-1/3}=\frac{1}{(-27)^{1/3}}=\frac{1}{\left(\sqrt[3]{-27}\right)}=\frac{1}{-3}=-\frac{1}{3}$

41. $(-8)^{-2/3}=\frac{1}{(-8)^{2/3}}=\frac{1}{\left(\sqrt[3]{(-8)^2}\right)}=\frac{1}{(-2)^2}=\frac{1}{4}$

42. $(-27)^{-2/3}=\frac{1}{(-27)^{2/3}}=\frac{1}{\left(\sqrt[3]{(-27)^2}\right)}=\frac{1}{(-3)^2}=\frac{1}{9}$

43. $\left(\frac{25}{16}\right)^{-1/2}=\left(\frac{16}{25}\right)^{1/2}=\left(\sqrt{\frac{16}{25}}\right)=\frac{4}{5}$

44. $\left(\frac{16}{9}\right)^{-1/2}=\left(\frac{9}{16}\right)^{1/2}=\left(\sqrt{\frac{9}{16}}\right)=\frac{3}{4}$

45. $\left(\frac{25}{16}\right)^{-3/2}=\left(\frac{16}{25}\right)^{3/2}=\left(\sqrt{\frac{16}{25}}\right)^3=\left(\frac{4}{5}\right)^3=\frac{64}{125}$

46. $\left(\frac{16}{9}\right)^{-3/2}=\left(\frac{9}{16}\right)^{3/2}=\left(\sqrt{\frac{9}{16}}\right)^3=\left(\frac{3}{4}\right)^3=\frac{27}{64}$

47. $\left(-\frac{1}{27}\right)^{-5/3}=(-27)^{5/3}=\left(\sqrt[3]{-27}\right)^5=(-3)^5=-243$

48. $\left(-\frac{1}{8}\right)^{-5/3}=(-8)^{5/3}=\left(\sqrt[3]{-8}\right)^5=(-2)^5=-32$

49. $7^{0.39}\approx 2.14$

50. $5^{0.47}\approx 2.13$

51. $8^{2.7}\approx 274.37$

52. $5^{3.9}\approx 532.09$

53. $\left[(0.1)^{0.1}\right]^{0.1}\approx 0.977$

54. $\left(1+\frac{1}{1000}\right)^{1000}\approx 2.717$

55. $\left(1-\frac{1}{1000}\right)^{-1000}\approx 2.720$

56. $\left(1+10^{-6}\right)^{10^6}\approx 2.718$

57. $\frac{4}{x^5}=4x^{-5}$

58. $\frac{6}{2x^3}=\frac{3}{x^3}=3x^{-3}$

59. $\frac{4}{\sqrt[3]{8x^3}}=\frac{4}{2x^{4/3}}=2x^{-4/3}$

60. $\frac{6}{\sqrt{4x^3}}=\frac{6}{2x^{3/2}}=3x^{-3/2}$

61. $\frac{24}{\left(2\sqrt{x}\right)^3}=\frac{24}{8x^{3/2}}=3x^{-3/2}$

62. $\frac{18}{\left(3\sqrt[3]{x}\right)^2}=\frac{18}{9x^{2/3}}=2x^{-2/3}$

63. $\sqrt{\frac{9}{x^4}}=\frac{3}{x^2}=3x^{-2}$

64. $\sqrt[3]{\frac{8}{x^6}}=\frac{2}{x^2}=2x^{-2}$

65. $\frac{5x^2}{\sqrt{x}}=\frac{5x^2}{x^{1/2}}=5x^{2-1/2}=5x^{3/2}$

66. $\frac{3\sqrt{x}}{x}=\frac{3x^{1/2}}{x}=3x^{1/2-1}=3x^{-1/2}$

67. $\frac{12\sqrt[3]{x^2}}{3x^2}=\frac{12x^{2/3}}{3x^2}=\frac{12}{3}x^{2/3-2}=4x^{-4/3}$

68. $\frac{10\sqrt{x}}{2\sqrt[3]{x}}=\frac{10x^{1/2}}{2x^{1/3}}=\frac{10}{2}x^{1/2-1/3}=5x^{1/6}$

69. $\frac{\sqrt{36x}}{2x}=\frac{6\sqrt{x}}{2x}=\frac{6x^{1/2}}{2x}=\frac{6}{2}x^{1/2-1}=3x^{-1/2}$

70. $\frac{\sqrt[3]{8x^2}}{4x}=\frac{2\sqrt[3]{x^2}}{4x}=\frac{2x^{2/3}}{4x}=\frac{2}{4}x^{2/3-1}=\frac{1}{2}x^{-1/3}$

71. $\left(x^3 \cdot x^2\right)^2 = \left(x^5\right)^2 = x^{10}$

72. $\left(x^4 \cdot x^3\right)^2 = \left(x^7\right)^2 = x^{14}$

73. $\left[z^2\left(z \cdot z^2\right)^2 z\right]^3 = \left[z^2\left(z^3\right)^2 z\right]^3 = \left(z^2 \cdot z^6 \cdot z\right)^3$
$= \left(z^9\right)^3 = z^{27}$

74. $\left[z\left(z^3 \cdot z\right)^2 z^2\right]^2 = \left[z\left(z^4\right)^2 z^2\right]^2 = \left(z \cdot z^8 \cdot z^2\right)^2$
$= \left(z^{11}\right)^2 = z^{22}$

75. $\left[\left(x^2\right)^2\right]^2 = \left(x^4\right)^2 = x^8$

76. $\left[\left(x^3\right)^3\right]^3 = \left(x^9\right)^3 = x^{27}$

77. $(3x^2y^5z)^3 = 3^3x^{2\cdot 3}y^{5\cdot 3}z^3 = 27x^6y^{15}z^3$

78. $(2x^4yz^6)^4 = 2^4x^{4\cdot 4}y^4z^{6\cdot 4} = 16x^{16}y^4z^{24}$

79. $\frac{(ww^2)^3}{w^3w} = \frac{(w^3)^3}{w^4} = \frac{w^9}{w^4} = w^5$

80. $\frac{(ww^3)^2}{w^3w^2} = \frac{(w^4)^2}{w^5} = \frac{w^8}{w^5} = w^3$

81. $\frac{\left(5xy^4\right)^2}{25x^3y^3} = \frac{25x^2y^8}{25x^3y^3} = \frac{y^5}{x}$

82. $\frac{\left(4x^3y\right)^2}{8x^2y^3} = \frac{16x^6y^2}{8x^2y^3} = \frac{2x^4}{y}$

83. $\frac{\left(9xy^3z\right)^2}{3\left(xyz\right)^2} = \frac{81x^2y^6z^2}{3x^2y^2z^2} = 27y^4$

84. $\frac{\left(5x^2y^3z\right)^2}{5\left(xyz\right)^2} = \frac{25x^4y^6z^2}{5x^2y^2z^2} = 5x^2y^4$

85. $\frac{\left(2u^2vw^3\right)^2}{4\left(uw^2\right)^2} = \frac{4u^4v^2w^6}{4u^2w^4} = u^2v^2w^2$

86. $\frac{\left(u^3vw^2\right)^2}{9\left(u^2w\right)^2} = \frac{u^6v^2w^4}{9u^4w^2} = \frac{u^2v^2w^2}{9}$

87. Average body thickness
$= 0.4(\text{hip-to-shoulder length})^{3/2}$
$= 0.4(16)^{3/2}$
$= 0.4\left(\sqrt{16}\right)^3$
≈ 25.6 ft

88. Average body thickness
$= 0.4(\text{hip-to-shoulder length})^{3/2}$
$= 0.4(14)^{3/2}$
$= 0.4\left(\sqrt{14}\right)^3$
≈ 21.0 ft

89. $C' = x^{0.6}C$
$= 4^{0.6}C \approx 2.3C$
To quadruple the capacity costs about 2.3 times as much.

90. $C' = x^{0.6}C$
$= 3^{0.6}C \approx 1.9C$
To triple the capacity costs about 1.9 times as much

91. **a.** Given the unemployment rate of 2 percent, the inflation rate is
$y = 9.638(2)^{-1.394} - 0.900$
≈ 2.77 percent.

b. Given the unemployment rate of 5 percent, the inflation rate is
$y = 9.638(5)^{-1.394} - 0.900$
≈ 0.12 percent.

92. **a.** Given the unemployment rate of 3 percent, the inflation rate is
$y = 45.4(3)^{-1.54} - 1$
≈ 7.36 percent.

b. Given the unemployment rate of 8 percent, the inflation rate is
$y = 45.4(8)^{-1.54} - 1$
≈ 0.85 percent.

93. $(\text{Heart rate}) = 250(\text{weight})^{-1/4}$
$= 250(16)^{-1/4}$
$= 125$ beats per minute

94. $(\text{Heart rate}) = 250(\text{weight})^{-1/4}$
$= 250(625)^{-1/4}$
$= 50$ beats per minute

95. (Time to build the 50th Boeing 707)
$=150(50)^{-0.322}$
≈ 42.6 thousand work-hours

It took approximately 42,600 work-hours to build the 50th Boeing 707.

96. (Time to build the 250th Boeing 707)
$=150(250)^{-0.322}$
≈ 25.3 thousand work-hours

It took approximately 25,300 work-hours to build the 250th Boeing 707.

97.
$$\begin{aligned}\text{Increase in energy} &= 32^{B-A}\\ &= 32^{7.8-6.7}\\ &= 32^{1.1} \approx 45\end{aligned}$$
The 1906 San Francisco earthquake had about 45 times more energy released than the 1994 Northridge earthquake.

98.
$$\begin{aligned}\text{Increase in energy} &= 32^{B-A}\\ &= 32^{9.0-7.7}\\ &= 32^{1.3} \approx 91\end{aligned}$$
The 2011 Japan earthquake had about 91 times more energy released than the 2011 India earthquake.

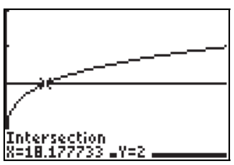
99. $K = 3000(225)^{-1/2} = 200$

100. $K = 4000(125)^{-2/3} = 160$

101.
$$\begin{aligned}S &= \frac{60}{11}x^{0.5}\\ &= \frac{60}{11}(3281)^{0.5} \approx 312 \text{ mph}\end{aligned}$$

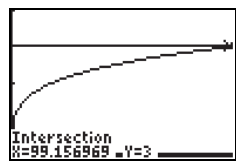
102.
$$\begin{aligned}S &= \frac{60}{11}x^{0.5}\\ &= \frac{60}{11}(1650)^{0.5} \approx 222 \text{ mph}\end{aligned}$$

103.



on [0, 100] by [0,4]
$x \approx 18.2$. Therefore, the land area must be increased by a factor of more than 18 to double the number of species.

104.

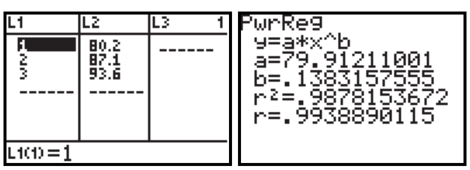


on [0, 100] by [0,4]
$x \approx 99$. Therefore, the land area must be increased by almost 100 times to triple the number of species.

105. $y = 9.4x^{0.37}$
$y = 9.4(150)^{0.37} = 60$ miles per hour
The speed of a car that left 150-foot skid marks was 60 miles per hour.

106. $y = 9.4(350)^{0.37} = 82$ miles per hour
The speed of a car that left 350-foot skid marks was 82 miles per hour.

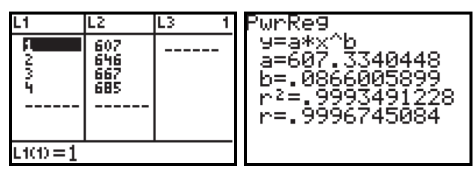
107. **a.**



$y = 79.9x^{0.138}$ (rounded)

b. For year 2020, $x = 10$.
$y = 79.9(10)^{0.138} \approx \$110$ billion

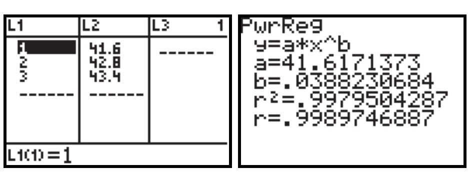
108. **a.**



$y = 607x^{0.0866}$ (rounded)

b. For year 2020, $x = 12$
$y = 607(12)^{0.0866} \approx \$753$

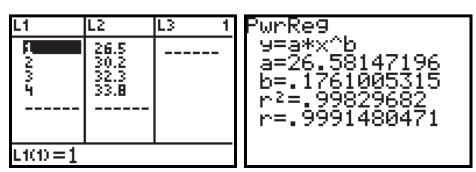
109. **a.**



$y = 41.6x^{0.0388}$ (rounded)

b. For year 2020, $x = 11$.
$y = 41.6(11)^{0.0388} \approx \$45.7$ million

110. **a.**



$y = 26.6x^{0.176}$ (rounded)

b. For year 2020, $x = 12$.
$y = 26.6(12)^{0.176} \approx \$41.2$ billion

111. 3, since $\sqrt{9}$ means the principal square foot. (To get ± 3 you would have to write $\pm\sqrt{9}$.)

112. False: $2^2 \cdot 2^3 = 4 \cdot 8 = 32$, while $2^{2\cdot 3} = 2^6 = 64$. (The correct statement is $x^m \cdot x^n = x^{m+n}$.

113. False: $\frac{2^6}{2^6} = \frac{64}{4} = 16$, while $2^{6/2} = 2^3 = 8$.

(The correct statement is $\frac{x^m}{x^n} = x^{m-n}$.)

114. False: $\left(2^3\right)^2 = 8^2 = 64$, while $2^{3^2} = 2^9 = 512$. (The correct statement is $\left(x^m\right)^n = x^{m\cdot n}$.

115. $x^{1/2} = \sqrt{x}$, so x must be nonnegative for the expression to be defined.

116. $x^{1/3} = \sqrt[3]{x}$, so all values of x. For example, $8^{1/3} = 2$ and $(-8)^{1/3} = -2$.

117. $x^{-1} = \frac{1}{x}$, so all values of x except 0, because you cannot divide by 0.

118. If the exponent $\frac{m}{n}$ is not fully reduced, it will indicate an even root of a negative number, which is not defined in the real number set.

EXERCISES 1.3

1. Yes

2. No

3. No

4. Yes

5. No

6. Yes

7. No

8. Yes

9. Domain = $\{x \mid x \le 0 \text{ or } x \ge 1\}$
Range = $\{y \mid y \ge -1\}$

10. Domain = $\{x \mid x \le -1 \text{ or } x \ge 0\}$
Range = $\{y \mid y \le 1\}$

11. **a.** $f(x) = \sqrt{x-1}$
$f(10) = \sqrt{10-1} = \sqrt{9} = 3$

b. Domain = $\{x \mid x \ge 1\}$ since $f(x) = \sqrt{x-1}$ is defined for all values of $x \ge 1$.

c. Range = $\{y \mid y \ge 0\}$

12. **a.** $f(x) = \sqrt{x-4}$
$f(40) = \sqrt{40-4} = \sqrt{36} = 6$

b. Domain = $\{x \mid x \ge 4\}$ since $f(x) = \sqrt{x-4}$ is defined for all values of $x \ge 4$.

c. Range = $\{y \mid y \ge 0\}$

13. **a.** $h(z) = \frac{1}{z+4}$
$h(-5) = \frac{1}{-5+4} = -1$

b. Domain $=\{z \mid z \ne -4\}$ since $h(z) = \frac{1}{z+4}$ is defined for all values of z except $z = -4$.

c. Range = $\{y \mid y \ne 0\}$

14. **a.** $h(z) = \frac{1}{z+7}$
$h(-8) = \frac{1}{-8+7} = -1$

b. Domain = $\{z \mid z \ne -7\}$ since $h(z) = \frac{1}{z+7}$ is defined for all values of z except $z = -7$.

c. Range = $\{y \mid y \ne 0\}$

15. **a.** $h(x) = x^{1/4}$
$h(81) = 81^{1/4} = \sqrt[4]{81} = 3$

b. Domain = $\{x \mid x \ge 0\}$ since $h(x) = x^{1/4}$ is defined only for nonnegative values of x.

c. Range = $\{y \mid y \ge 0\}$

16. **a.** $h(x) = x^{1/6}$
$h(81) = 64^{1/4} = \sqrt[6]{64} = 2$

b. Domain = $\{x \mid x \ge 0\}$ since $h(x) = x^{1/6}$ is defined for nonnegative values of x.

c. Range = $\{y \mid y \ge 0\}$

17. **a.** $f(x) = x^{2/3}$
$f(-8) = (-8)^{2/3} = \left(\sqrt[3]{-8}\right)^2 = (-2)^2 = 4$

b. Domain = $\mathbb{R}$

c. Range = $\{y \mid y \ge 0\}$

18. **a.** $f(x) = x^{4/5}$
$f(-32) = (-32)^{4/5} = \left(\sqrt[5]{-32}\right)^4 = (-2)^4 = 16$

b. Domain = $\mathbb{R}$

c. Range = $\{y \mid y \ge 0\}$

19. **a.** $f(x)=\sqrt{4-x^2}$
$f(0)=\sqrt{4-0^2}=\sqrt{4}=2$

b. $f(x)=\sqrt{4-x^2}$ is defined for values of x such that $4-x^2\geq 0$. Thus,
$4-x^2\geq 0$
$-x^2\geq -4$
$x^2\leq 4$
$-2\leq x\leq 2$
Domain = $\{x \mid -2\leq x\leq 2\}$

c. Range = $\{y \mid 0\leq y\leq 2\}$

20. **a.** $f(x)=\dfrac{1}{\sqrt{x}}$
$f(4)=\dfrac{1}{\sqrt{4}}=\dfrac{1}{2}$

b. Domain = $\{x \mid x>0\}$ since $f(x)=\dfrac{1}{\sqrt{x}}$ is defined only for positive values of x.

c. Range = $\{y \mid y>0\}$

21. **a.** $f(x)=\sqrt{-x}$
$f(-25)=\sqrt{-(-25)}=\sqrt{25}=5$

b. $f(x)=\sqrt{-x}$ is defined only for values of x such that $-x\geq 0$. Thus $x\leq 0$.
Domain = $\{x \mid x\leq 0\}$

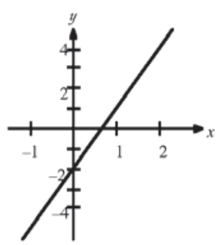
c. Range = $\{y \mid y\geq 0\}$

22. **a.** $f(x)=-\sqrt{-x}$
$f(-100)=-\sqrt{-(-100)}=-\sqrt{100}=-10$

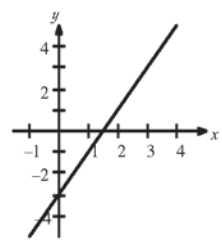
b. $f(x)=-\sqrt{-x}$ is defined only for values of x such that $-x\geq 0$. Thus $x\leq 0$.
Domain = $\{x \mid x\leq 0\}$

c. Range = $\{y \mid y\leq 0\}$

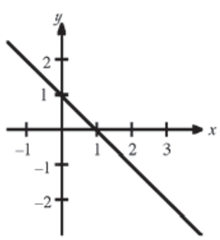
23.



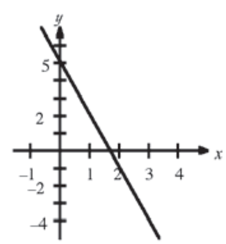
24.



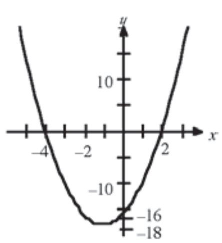
25.



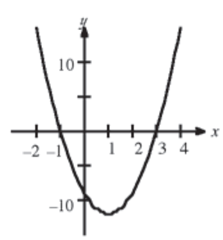
26.



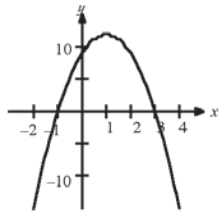
27.



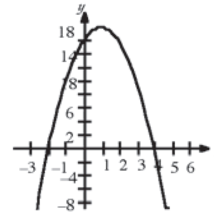
28.



29.



30.



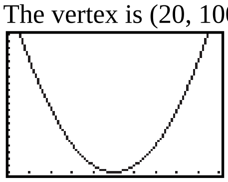
31. **a.** $x = \frac{-b}{2a} = \frac{-(-40)}{2(1)} = \frac{40}{2} = 20$

To find the y-coordinate, evaluate f at $x = 20$.

$f(20) = (20)^2 - 40(20) + 500 = 100$

The vertex is (20, 100).

b.



on [15, 25] by [100, 120]

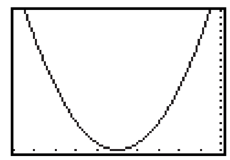
32. **a.** $x = \frac{-b}{2a} = \frac{-40}{2(1)} = \frac{-40}{2} = -20$

To find the y-coordinate, evaluate f at $x = -20$.

$f(-20) = (-20)^2 + 40(-20) + 500 = 100$

The vertex is (–20, 100).

b.



on [–25, –15] by [100, 120]

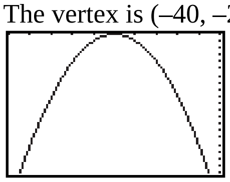
33. **a.** $x = \frac{-b}{2a} = \frac{-(-80)}{2(-1)} = \frac{80}{-2} = -40$

To find the y-coordinate, evaluate f at $x = -40$.

$$\begin{aligned} f(-40) &= -(-40)^2 - 80(-40) - 1800 \\ &= -200 \end{aligned}$$

The vertex is (–40, –200).

b.



on [–45, –35] by [–220, –200]

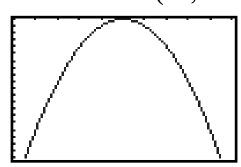
34. **a.** $x = \frac{-b}{2a} = \frac{-80}{2(-1)} = \frac{-80}{-2} = 40$

To find the y-coordinate, evaluate f at $x = 40$.

$$\begin{aligned} f(-40) &= -(40)^2 + 80(40) - 1800 \\ &= -200 \end{aligned}$$

The vertex is (40, –200).

b.



on [35, 45] by [–220, –200]

35.

$$x^2 - 6x - 7 = 0$$
$$(x-7)(x+1) = 0$$

Equals 0 at $x = 7$; Equals 0 at $x = -1$

$x = 7, \quad x = -1$

36.

$$x^2 - x - 20 = 0$$
$$(x-5)(x+4) = 0$$

Equals 0 at $x = 5$; Equals 0 at $x = -4$

$x = 5, \quad x = -4$

37.

$$x^2 + 2x = 15$$
$$x^2 + 2x - 15 = 0$$
$$(x+5)(x-3) = 0$$

Equals 0 at $x = -5$; Equals 0 at $x = 3$

$x = -5, \quad x = 3$

38.

$$x^2 - 3x = 54$$
$$x^2 - 3x - 54 = 0$$
$$(x-9)(x+6) = 0$$

Equals 0 at $x = 9$; Equals 0 at $x = -6$

$x = 9, \quad x = -6$

39.
$$2x^2 + 40 = 18x$$
$$2x^2 - 18x + 40 = 0$$
$$x^2 - 9x + 20 = 0$$
$$(x-4)(x-5) = 0$$
Equals 0 at $x = 4$; Equals 0 at $x = 5$

$x = 4$, $x = 5$

40.
$$3x^2 + 18 = 15x$$
$$3x^2 - 15x + 18 = 0$$
$$x^2 - 5x + 6 = 0$$
$$(x-3)(x-2) = 0$$
Equals 0 at $x = 3$; Equals 0 at $x = 2$

$x = 3$, $x = 2$

41.
$$5x^2 - 50x = 0$$
$$x^2 - 10x = 0$$
$$x(x-10) = 0$$
Equals 0 at $x=0$; Equals 0 at $x=10$

$x=0$, $x=10$

42.
$$3x^2 - 36x = 0$$
$$x^2 - 12x = 0$$
$$x(x-12) = 0$$
Equals 0 at $x=0$; Equals 0 at $x=12$

$x=0$, $x=12$

43.
$$2x^2 - 50 = 0$$
$$x^2 - 25 = 0$$
$$(x-5)(x+5) = 0$$
Equals 0 at $x = 5$; Equals 0 at $x = -5$

$x = 5$, $x = -5$

44.
$$3x^2 - 27 = 0$$
$$x^2 - 9 = 0$$
$$(x-3)(x+3) = 0$$
Equals 0 at $x = 3$; Equals 0 at $x = -3$

$x = 3$, $x = -3$

45.
$$4x^2+24x+40=4$$
$$4x^2+24x+36=0$$
$$x^2+6x+9=0$$
$$(x+3)^2=0$$
Equals 0 at $x=-3$

46.
$$3x^2-6x+9=6$$
$$3x^2-6x+3=0$$
$$x^2-2x+1=0$$
$$(x-1)^2=0$$
Equals 0 at $x=1$

47.
$$-4x^2 + 12x = 8$$
$$-4x^2 + 12x - 8 = 0$$
$$x^2 - 3x + 2 = 0$$
$$(x-2)(x-1) = 0$$
Equals 0 at $x = 2$; Equals 0 at $x = 1$

$x = 2$, $x = 1$

48.
$$-3x^2 + 6x = -24$$
$$-3x^2 + 6x + 24 = 0$$
$$x^2 - 2x - 8 = 0$$
$$(x-4)(x+2) = 0$$
Equals 0 at $x = 4$; Equals 0 at $x = -2$

$x = 4$, $x = -2$

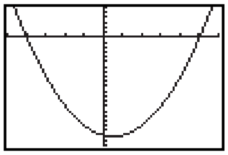
49.
$$2x^2 - 12x + 20 = 0$$
$$x^2 - 6x + 10 = 0$$
Use the quadratic formula with $a = 1$, $b = -6$, and $c = 10$.
$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(10)}}{2(1)}$$
$$= \frac{6 \pm \sqrt{36-40}}{2}$$
$$= \frac{6 \pm \sqrt{-4}}{2} \quad \text{Undefined}$$
$2x^2 - 12x + 20 = 0$ has no real solutions.

50.
$$2x^2 - 8x + 10 = 0$$
$$x^2 - 4x + 5 = 0$$
Use the quadratic formula with $a = 1$, $b = -4$, and $c = 5$.
$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$$
$$= \frac{4 \pm \sqrt{16-20}}{2}$$
$$= \frac{4 \pm \sqrt{-4}}{2} \quad \text{Undefined}$$
$2x^2 - 8x + 10 = 0$ has no real solutions.

51.
$$3x^2 + 12 = 0$$
$$x^2 + 4 = 0$$
$$x^2 = -4$$
$$x = \pm\sqrt{-4} \quad \text{Undefined}$$
$3x^2 + 12 = 0$ has no real solutions.

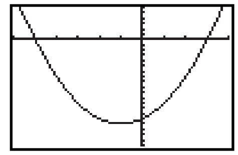
52.
$$5x^2 + 20 = 0$$
$$x^2 + 4 = 0$$
$$x^2 = -4$$
$$x = \pm\sqrt{-4} \quad \text{Undefined}$$
$5x^2 + 20 = 0$ has no real solutions.

53.



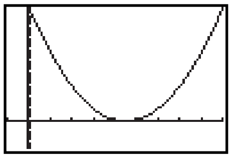
on [–5, 6] by [–22, 6]
$x = -4$, $x = 5$

54.



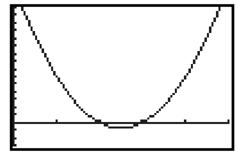
on [–6, 4] by [–20, 6]
$x = -5$, $x = 3$

55.



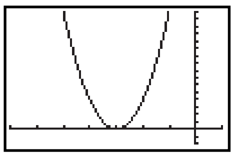
on [–1, 9] by [–10, 40]
$x = 4$, $x = 5$

56.



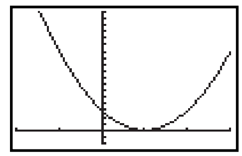
on [0, 5] by [–3, 15]
$x = 2$, $x = 3$

57.



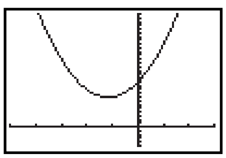
on [–7, 1] by [–2, 16]
$x = -3$

58.



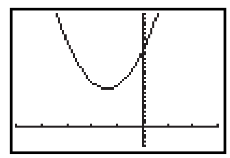
on [–2, 3] by [–2, 18]
$x = 1$

59.



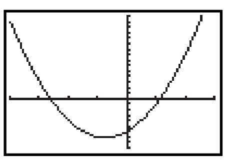
on [–5, 3] by [–5, 30]
No real solutions

60.



on [–5, 3] by [–5, 30]
No real solutions

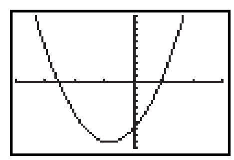
61.



on [–4, 3] by [–9, 15]

$x = -2.64$, $x = 1.14$

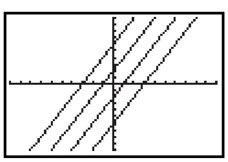
62.



on [–4, 3] by [–10, 10]

$x = -2.57$, $x = 0.91$

63.

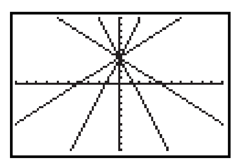


on [–10, 10] by [–10, 10]

a. Their slopes are all 2, but they have different y-intercepts.

b. The line 2 units below the line of the equation $y = 2x - 6$ must have y-intercept –8. Thus, the equation of this line is $y = 2x - 8$.

64.



on [–10, 10] by [–10, 10]

a. The lines have the same y-intercept, but their slopes are different.

b. $y = \frac{1}{2}x + 4$

65. Let x = the number of board feet of wood. Then $C(x) = 4x + 20$

66. Let x = the number of bicycles. Then $C(x) = 55x + 900$

67. Let x = the number of hours of overtime. Then P(x) = 15x + 500

68. Let x = the total week's sales. Then P(x) = 0.02x + 300

69. **a.**
$$\begin{aligned} p(d) &= 0.45d + 15 \\ p(6) &= 0.45(6) + 15 \\ &= 17.7 \text{ pounds per square inch} \end{aligned}$$

b.
$$\begin{aligned} p(d) &= 0.45d + 15 \\ p(35{,}000) &= 0.45(35{,}000) + 15 \\ &= 15{,}765 \text{ pounds per square inch} \end{aligned}$$

70.
$$\begin{aligned} B(h) &= -1.8h + 212 \\ 98.6 &= -1.8h + 212 \\ 1.8h &= 113.4 \\ h &= 63 \text{ thousand feet above sea level} \end{aligned}$$

71.
$$\begin{aligned} D(v) &= 0.055v^2 + 1.1v \\ D(40) &= 0.055(40)^2 + 1.1(40) = 132 \text{ ft} \end{aligned}$$

72.
$$\begin{aligned} D(v) &= 0.55v^2 + 1.1v \\ D(60) &= 0.55(60)^2 + 1.1(60) = 264 \text{ ft} \end{aligned}$$

73. **a.**
$$\begin{aligned} N(t) &= 200 + 50t^2 \\ N(2) &= 200 + 50(2)^2 \\ &= 400 \text{ cells} \end{aligned}$$

b.
$$\begin{aligned} N(t) &= 200 + 50t^2 \\ N(10) &= 200 + 50(10)^2 \\ &= 5200 \text{ cells} \end{aligned}$$

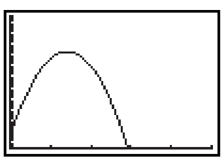
74. **a.**
$$\begin{aligned} T(h) &= 0.5\sqrt{h} \\ T(4) &= 0.5\sqrt{4} = 1 \text{ second} \\ T(8) &= 0.5\sqrt{8} \approx 1.4 \text{ seconds} \end{aligned}$$

b.
$$\begin{aligned} T(h) &= 0.5\sqrt{h} & T(h) &= 0.5\sqrt{h} \\ 2 &= 0.5\sqrt{h} & 3 &= 0.5\sqrt{h} \\ 4 &= \sqrt{h} & 6 &= \sqrt{h} \\ h &= 16 \text{ ft} & h &= 36 \text{ ft} \end{aligned}$$

75.
$$\begin{aligned} v(x) &= \frac{60}{11}\sqrt{x} \\ v(1776) &= \frac{60}{11}\sqrt{1776} \approx 230 \text{ mph} \end{aligned}$$

76.
$$\begin{aligned} s(d) &= 3.86\sqrt{d} \\ s(15{,}000) &= 3.86\sqrt{15{,}000} \approx 473 \text{ mph} \end{aligned}$$

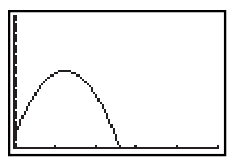
77.



on [0, 5] by [0, 50]

The object hits the ground in about 2.92 seconds

78.



on [0, 5] by [0, 50]

The object hits the ground in about 2.6 seconds.

79. **a.** To find the break-even points, set $C(x)$ equal to $R(x)$ and solve the resulting equation.

$$\begin{aligned} C(x) &= R(x) \\ 180x + 16{,}000 &= -2x^2 + 660x \\ 2x^2 - 480x + 16{,}000 &= 0 \end{aligned}$$

Use the quadratic formula with $a = 2$, $b = -480$ and $c = 16{,}000$.

$$x = \frac{480 \pm \sqrt{(-480)^2 - 4(2)(16{,}000)}}{2(2)}$$

$$x = \frac{480 \pm \sqrt{102{,}400}}{4} = \frac{480 \pm 320}{4}$$

$$x = \frac{800}{4} \text{ or } \frac{160}{4}$$

$$x = 200 \text{ or } 40$$

The company will break even when it makes either 40 devices or 200 devices.

b. To find the number of devices that maximizes profit, first find the profit function, $P(x) = R(x) - C(x)$.

$$\begin{aligned} P(x) &= (-2x^2 + 660x) - (180x + 16{,}000) \\ &= -2x^2 + 480x - 16{,}000 \end{aligned}$$

Since this is a parabola that opens downward, the maximum profit is found at the vertex.

$$x = \frac{-480}{2(-2)} = \frac{-480}{-4} = 120$$

Thus, profit is maximized when 120 devices are produced per week. The maximum profit is found by evaluating $P(120)$.

$$\begin{aligned} P(120) &= -2(120)^2 + 480(120) - 16{,}000 \\ &= \$12{,}800 \end{aligned}$$

Therefore, the maximum profit is \$12,800.

80. **a.** To find the break-even points, set $C(x)$ equal to $R(x)$ and solve the resulting equation.

$$C(x)=R(x)$$
$$420x+72,000=-3x^2+1800x$$
$$3x^2-1380x+72,000=0$$

Use the quadratic formula with a = 3, b = –1380 and c = 72,000.

$$x=\frac{1380\pm\sqrt{(-1380)^2-4(3)(72,000)}}{2(3)}$$
$$x=\frac{1380\pm\sqrt{1,040,400}}{6}=\frac{1380\pm1020}{6}$$
$$x=\frac{2400}{6} \text{ or } \frac{360}{6}$$
$$x=400 \text{ or } 60$$

The store will break even when it sells either 60 bicycles or 400 bicycles.

b. To find the number of bicycles that maximizes profit, first find the profit function, $P(x) = R(x) - C(x)$.

$$P(x)=(-3x^2+1800x)-(420x+72,000)$$
$$=-3x^2+1380x-72,000$$

Since this is a parabola that opens downward, the maximum profit is found at the vertex.

$$x = \frac{-1380}{2(-3)} = \frac{-1380}{-6} = 230$$

Thus, profit is maximized when 230 bicycles are sold per month. The maximum profit is found by evaluating $P(230)$.

$$P(230)=-3(230)^2+1380(230)-72,000$$
$$=\$86,700$$

Therefore, the maximum profit is \$86,700.

81. **a.** To find the break-even points, set $C(x)$ equal to $R(x)$ and solve the resulting equation.

$$C(x)=R(x)$$
$$100x+3200=-2x^2+300x$$
$$2x^2-200x+3200=0$$

Use the quadratic formula with a = 2, b = –200 and c = 3200.

$$x=\frac{200\pm\sqrt{(-1020)^2-4(2)(3200)}}{2(2)}$$
$$x=\frac{200\pm\sqrt{14,400}}{4}=\frac{200\pm120}{4}$$
$$x=\frac{320}{4} \text{ or } \frac{80}{4}$$
$$x=80 \text{ or } 20$$

The store will break even when it sells either 20 exercise machines or 80 exercise machines.

b. To find the number of exercise machines that maximizes profit, first find the profit function, $P(x) = R(x) - C(x)$.

$$P(x) = (-2x^2+300x)-(100x+3200)$$
$$= -2x^2+200x-3200$$

Since this is a parabola that opens downward, the maximum profit is found at the vertex.

$$x = \frac{-200}{2(-2)} = \frac{-200}{-4} = 50$$

Thus, profit is maximized when 50 exercise machines are sold per day. The maximum profit is found by evaluating $P(50)$.

$$P(50) = -2(50)^2 + 200(50) - 3200$$
$$= \$1800$$

Therefore, the maximum profit is \$1800.

82. Since this is a parabola that opens downward, the monthly price that maximizes visits is found at the vertex.

$$x=\frac{-0.56}{2(-0.004)}=\$70$$

83.

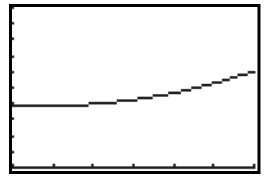
$$(w+a)(v+b)=c$$
$$v+b=\frac{c}{w+a}$$
$$v=\frac{c}{w+a}-b$$

84. **a.** $f(1) = -0.077(1)^2 - 0.057(1) + 1$
$= -0.077 - 0.057 + 1$
$= 0.866$
So a 65-year-old person has an 86.6% chance of living another decade.

b. $f(2) = -0.077(2)^2 - 0.057(2) + 1$
$= -0.308 - 0.114 + 1$
$= 0.578$
So a 65-year-old person has a 57.8% chance of living two more decades.

c. $f(3) = -0.077(3)^2 - 0.057(3) + 1$
$= -0.693 - 0.171 + 1$
$= 0.136$
So a 65-year-old person has a 13.6% chance of living three more decades.

85. **a.**

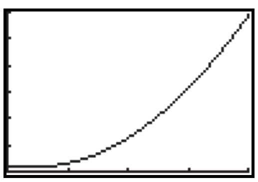


On [10, 16] by [0, 100].

b. $y = 0.831x^2 - 18.1x + 137.3$
$y = 0.831(12)^2 - 18.1(12) + 137.3$
$y = 39.764$
The probability that a high school graduate smoker will quit is 40%.

c. $y = 0.831x^2 - 18.1x + 137.3$
$y = 0.831(16)^2 - 18.1(16) + 137.3$
$y = 60.436$
The probability that a college graduate smoker will quit is 60%.

86. **a.**



On [0, 20] by [0, 300].

b. 2015 – 1995 = 20
$y = 0.9x^2 - 3.9x + 12.4$
$y = 0.9(20)^2 - 3.9(20) + 12.4$
$y = 294.4$
So the global wind power generating capacity in the year 2015 is about 294 thousand megawatts.

c. 2020 – 1995 = 25
$y = 0.9x^2 - 3.9x + 12.4$
$y = 0.9(25)^2 - 3.9(25) + 12.4$
$y = 477.4$
So the global wind power generating capacity in the year 2020 is about 477 thousand megawatts.

87. **a.** $(100 - x)x = 100x - x^2$ or $-x^2 + 100$
$f(x) = 100x - x^2$ or $f(x) = -x^2 + 100x$

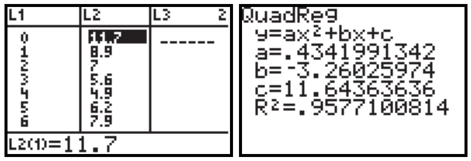
b. Since this function represents a parabola opening downward (because $a = -1$), it is maximized at its vertex, which is found using the vertex formula, $x = \frac{-b}{2a}$, with $a = -1$ and $b = 100$.

$x = \frac{-100}{-2} = 50$
She should charge $50 to maximize her revenue.

88. **a.** The upper limit is
$f(x) = 0.7(220 - x) = 154 - 0.7x$
The lower limit is
$f(x) = 0.5(220 - x) = 110 - 0.5x$

b. The lower cardio limit for a 20-year old is
$g(20) = 110 - 0.5(20) = 100$ bpm
The upper cardio limit for a 20-year old is
$f(20) = 154 - 0.7(20) = 140$ bpm
The lower cardio limit for a 60-year old is
$g(60) = 110 - 0.5(60) = 80$ bpm
The upper cardio limit for a 60-year old is
$f(60) = 154 - 0.7(60) = 112$ bpm

89. **a.**

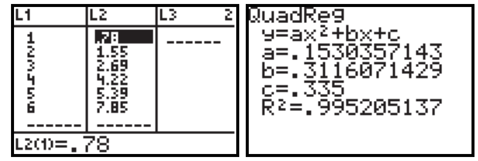


$y = 0.434x^2 - 3.26x + 11.6$

b. For year 2016, x = 8.6.

$y = 0.434(8.6)^2 - 3.26(8.6) + 11.6 \approx 15.7\%$

90. **a.**



$y = 0.153x^2 + 0.312x + 0.335$

b. For year 2020, x = 7.

$y = 0.153(7)^2 + 0.312(7) + 0.335 \approx \$10$

For year 2030, x = 8.

$y = 0.153(8)^2 + 0.312(8) + 0.335 \approx \$12.60$

91. A function can have more than one x-intercept. Many parabolas cross the x-axis twice. A function cannot have more than one y-intercept because that would violate the vertical line test.

92. $f(4) = 9$, (since the two given values show that x increasing by 1 means y increases by 2.).

93. Because the function is linear and 5 is halfway between 4 and 6, $f(5) = 9$ (halfway between 7 and 11).

94. The units of $f(x)$ is widgets and the units of x are blivets, so the units of the slope would be widgets per blivet.

95. m is blargs per prendle and $\dfrac{\Delta y}{\Delta x}$, so x is in prendles and y is in blargs.

96. If a is negative, then it will have a vertex that is its highest value. If a is positive, then the equation will have a vertex that is its lowest value.

97. No, that would violate the vertical line test. Note: A *parabola* is a geometric shape and so *may* open sideways, but a quadratic function, being a *function*, must pass the vertical line test.

98. Either by the symmetry of parabolas, or, better, by taking the average of the two x-intercepts: the $\pm$ part of the quadratic formula will cancel out, leaving just $\dfrac{-b}{2a}$.

EXERCISES 1.4

1. Domain = $\{x \mid x < -4 \text{ or } x > 0\}$
Range = $\{y \mid y < -2 \text{ or } y > 0\}$

2. Domain = $\{x \mid x \le 0 \text{ or } x > 3\}$
Range = $\{y \mid y \le -2 \text{ or } y > 2\}$

3. **a.** $f(x) = \dfrac{1}{x+4}$

$f(-3) = \dfrac{1}{-3+4} = 1$

b. Domain = $\{x \mid x \ne -4\}$

c. Range = $\{y \mid y \ne 0\}$

4. **a.** $f(x) = \dfrac{1}{(x-1)^2}$

$f(-1) = \dfrac{1}{(-1-1)^2} = \dfrac{1}{4}$

b. Domain = $\{x \mid x \ne 1\}$

c. Range = $\{y \mid y > 0\}$

5. **a.** $f(x) = \dfrac{x^2}{x-1}$

$f(-1) = \dfrac{(-1)^2}{-1-1} = -\dfrac{1}{2}$

b. Domain = $\{x \mid x \ne 1\}$

c. Range = $\{y \mid y \le 0 \text{ or } y \ge 4\}$

6. **a.** $f(x) = \dfrac{x^2}{x+2}$

$f(2) = \dfrac{2^2}{2+2} = 1$

b. Domain = $\{x \mid x \ne 2\}$

c. Range = $\{y \mid y \le -8 \text{ or } y \ge 0\}$

7. **a.** $f(x) = \dfrac{12}{x(x+4)}$

$f(2) = \dfrac{12}{2(2+4)} = 1$

b. Domain = $\{x \mid x \neq 0, x \neq -4\}$

c. Range = $\{y \mid y \leq -3 \text{ or } y > 0\}$

8. **a.** $f(x) = \dfrac{16}{x(x-4)}$

$f(-4) = \dfrac{16}{-4(-4-4)} = \dfrac{1}{2}$

b. Domain = $\{x \mid x \neq 0, x \neq 4\}$

c. Range = $\{y \mid y \leq -4 \text{ or } y > 0\}$

9. **a.** $g(x) = |x+2|$

$g(-5) = |(-5)+2| = |-3| = 3$

b. Domain = $\mathbb{R}$

c. Range = $\{y \mid y \geq 0\}$

10. **a.** $g(x) = |x| + 2$

$g(-5) = |-5| + 2 = 5 + 2 = 7$

b. Domain = $\mathbb{R}$

c. Range = $\{y \mid y \geq 2\}$

11.

$$x^5 + 2x^4 - 3x^3 = 0$$
$$x^3(x^2 + 2x - 3) = 0$$
$$x^3(x+3)(x-1) = 0$$

Equals 0 at $x = 0$; Equals 0 at $x = -3$; Equals 0 at $x = 1$

$x = 0, x = -3,$ and $x = 1$

12.

$$x^6 - x^5 - 6x^4 = 0$$
$$x^4(x^2 - x - 6) = 0$$
$$x^4(x-3)(x+2) = 0$$

Equals 0 at $x = 0$; Equals 0 at $x = 3$; Equals 0 at $x = -2$

$x = 0, x = 3,$ and $x = -2$

13.

$$5x^3 - 20x = 0$$
$$5x(x^2 - 4) = 0$$
$$5x(x-2)(x+2) = 0$$

Equals 0 at $x = 0$; Equals 0 at $x = 2$; Equals 0 at $x = -2$

$x = 0, x = 2,$ and $x = -2$

14.

$$2x^5 - 50x^3 = 0$$
$$2x^3(x^2 - 25) = 0$$
$$2x^3(x-5)(x+5) = 0$$

Equals 0 at $x = 0$; Equals 0 at $x = 5$; Equals 0 at $x = -5$

$x = 0, x = 5,$ and $x = -5$

15.

$$2x^3 + 18x = 12x^2$$
$$2x^3 - 12x^2 + 18x = 0$$
$$2x(x^2 - 6x + 9) = 0$$
$$2x(x-3)^3 = 0$$

Equals 0 at $x = 0$; Equals 0 at $x = 3$

$x = 0$ and $x = 3$

16.

$$3x^3 + 12x^2 = 12x^3$$
$$3x^4 - 12x^3 + 12x^2 = 0$$
$$3x^2(x^2 - 4x + 4) = 0$$
$$3x^2(x-2)^2 = 0$$

Equals 0 at $x = 0$; Equals 0 at $x = 2$

$x = 0$ and $x = 2$

17.

$$6x^5 = 30x^4$$
$$6x^5 - 30x^4 = 0$$
$$6x^4(x-5) = 0$$

Equals 0 at $x = 0$; Equals 0 at $x = 5$

$x = 0$ and $x = 5$

18.

$$5x^4 = 20x^3$$
$$5x^4 - 20x^3 = 0$$
$$5x^3(x-4) = 0$$

Equals 0 at $x = 0$; Equals 0 at $x = 4$

$x = 0$ and $x = 4$

19.

$$3x^{5/2} - 6x^{3/2} = 9x^{1/2}$$
$$3x^{5/2} - 6x^{3/2} - 9x^{1/2} = 0$$
$$3x^{1/2}(x^2 - 2x - 3) = 0$$
$$3x^{1/2}(x-3)(x+1) = 0$$

Equals 0 at $x = 0$; Equals 0 at $x = 3$; Equals 0 at $\cancel{x = -1}$

$x = 0$, $x = 3$ and $\cancel{x = -1}$

Valid solutions are $x = 0$ and $x = 3$.

20.

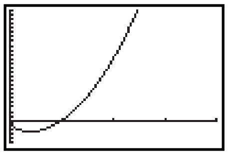
$$2x^{7/2} + 8x^{5/2} = 24x^{3/2}$$
$$2x^{7/2} + 8x^{5/2} - 24x^{3/2} = 0$$
$$2x^{3/2}(x^2 + 4x - 12) = 0$$
$$2x^{3/2}(x+6)(x-2) = 0$$

Equals 0 at $x = 0$; Equals 0 at $\cancel{x = -6}$; Equals 0 at $x = 2$

$x = 0$, $\cancel{x = -6}$ and $x = 2$

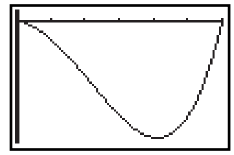
Valid solutions are $x = 0$ and $x = 2$.

21.



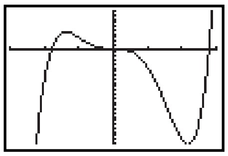
on [0, 4] by [–5,25]
$x = 0$ and $x = 1$

22.



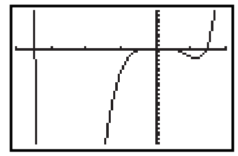
on [0, 6] by [–300, 25]
$x = 0$ and $x = 6$

23.



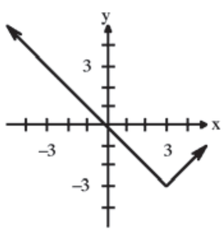
on [–3, 3] by [–25, 10]
$x \approx -1.79$, $x = 0$, and $x \approx 2.79$

24.

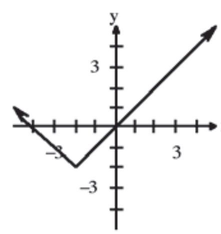


on [–4,2] by [–25, 10]
$x \approx -3.45$, $x = 0$, and $x \approx 1.45$

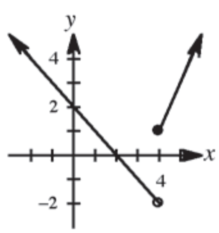
25.



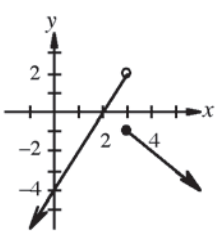
26.



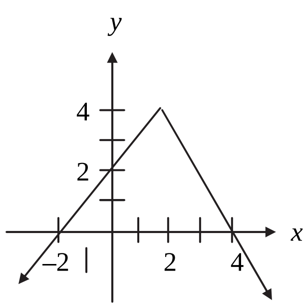
27.



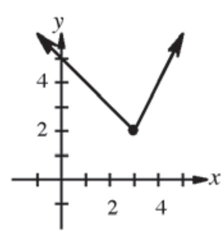
28.



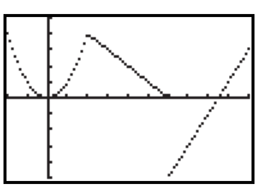
29.



30.

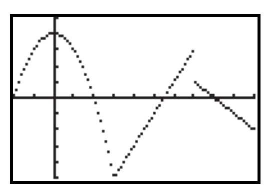


31.



on [–2, 10] by [–5, 5]

32.



on [–2, 10] by [–5, 5]

33. Polynomial
34. Piecewise linear function
35. Piecewise linear function
36. Polynomial
37. Polynomial
38. Piecewise linear function
39. Rational function
40. Polynomial

41. Piecewise linear function

42. Rational function

43. Polynomial

44. None of these

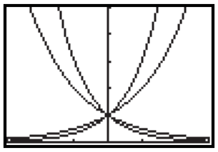
45. None of these

46. Polynomial

47. **a.** y_4

b. y_1

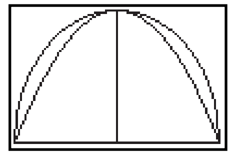
c.



on [–3, 3] by [0, 5]

d. (0, 1) because $a^0 = 1$ for any constant $a \neq 0$.

48.



on [–1, 1] by [0, 1]
The parabola is inside and the semicircle is outside.

49. **a.** $f(g(x)) = [g(x)]^5 = (7x-1)^5$

b. $g(f(x)) = 7[f(x)] - 1 = 7(x^5) - 1 = 7x^5 - 1$

50. **a.** $f(g(x)) = [g(x)]^8 = (2x+5)^8$

b. $g(f(x)) = 2[f(x)] + 5 = 2x^8 + 5$

51. **a.** $f(g(x)) = \dfrac{1}{g(x)} = \dfrac{1}{x^2+1}$

b. $g(f(x)) = [f(x)]^2 + 1 = \dfrac{1}{x^2} + 1$

52. **a.** $f(g(x)) = \sqrt{g(x)} = \sqrt{x^3 - 1}$

b. $g(f(x)) = [f(x)]^3 - 1 = \left(\sqrt{x}\right)^3 - 1 = x^{3/2} - 1$

53. **a.** $f(g(x)) = [g(x)]^3 - [g(x)]^2 = (\sqrt{x} - 1)^3 - (\sqrt{x} - 1)^2$

b. $g(f(x)) = \sqrt{f(x)} - 1 = \sqrt{x^3 - x^2} - 1$

54. **a.** $f(g(x)) = g(x) - \sqrt{g(x)} = x^2 + 1 - \sqrt{x^2 + 1}$

b. $g(f(x)) = [f(x)]^2 + 1 = (x - \sqrt{x})^2 + 1$

55. **a.** $f(g(x)) = \dfrac{[g(x)]^3 - 1}{[g(x)]^3 + 1} = \dfrac{(x^2 - x)^3 - 1}{(x^2 - x)^3 + 1}$

b. $g(f(x)) = [f(x)]^2 - f(x) = \left(\dfrac{x^3 - 1}{x^3 + 1}\right)^2 - \dfrac{x^3 - 1}{x^3 + 1}$

56. **a.** $f(g(x)) = \dfrac{[g(x)]^4 + 1}{[g(x)]^4 - 1} = \dfrac{(x^3 + x)^4 + 1}{(x^3 + x)^4 - 1}$

b. $g(f(x)) = [f(x)]^3 + f(x) = \left(\dfrac{x^4 + 1}{x^4 - 1}\right)^3 + \dfrac{x^4 + 1}{x^4 - 1}$

57. **a.** $f(g(x)) = 2[g(x)] - 6 = 2\left(\dfrac{x}{2} + 3\right) - 6 = x + 6 - 6$

b. $g(f(x)) = \dfrac{f(x)}{2} + 3 = \dfrac{2x - 6}{2} + 3 = x - 3 + 3 = x$

58. **a.** $f(g(x)) = [g(x)]^3 + 1 = \left(\sqrt[3]{x-1}\right)^3 + 1 = x - 1 + 1 = x$

b. $g(f(x)) = \sqrt[3]{(f(x)) - 1} = \sqrt[3]{x^3 + 1 - 1} = \sqrt[3]{x^3} = x$

59. $f(x+h)=5(x+h)^2=5(x^2+2hx+h^2)$
$=5x^2+10hx+5h^2$

60. $f(x+h)=3(x+h)^2=3(x^2+2hx+h^2)$
$=3x^2+6hx+3h^2$

61. $f(x+h)=2(x+h)^2-5(x+h)+1$
$=2x^2+4xh+2h^2-5x-5h+1$

62. $f(x+h)=3(x+h)^2-5(x+h)+2$
$=3x^2+6hx+3h^2-5x-5h+2$

63. $f(x)=5x^2$

$$\begin{aligned}\frac{f(x+h)-f(x)}{h}&=\frac{5(x+h)^2-5x^2}{h}\\&=\frac{5(x^2+2xh+h^2)-5x^2}{h}\\&=\frac{5x^2+10xh+5h^2-5x^2}{h}\\&=\frac{10xh+5h^2}{h}\\&=\frac{h(10x+5h)}{h}\\&=10x+5h \text{ or } 5(2x+h)\end{aligned}$$

64. $f(x)=3x^2$

$$\begin{aligned}\frac{f(x+h)-f(x)}{h}&=\frac{3(x+h)^2-3x^2}{h}\\&=\frac{3(x^2+2xh+h^2)-3x^2}{h}\\&=\frac{3x^2+6xh+3h^2-3x^2}{h}\\&=\frac{6xh+3h^2}{h}\\&=\frac{h(6x+3h)}{h}\\&=6x+3h \text{ or } 3(2x+h)\end{aligned}$$

65. $f(x)=2x^2-5x+1$

$$\begin{aligned}\frac{f(x+h)-f(x)}{h}&=\frac{2(x+h)^2-5(x+h)+1-(2x^2-5x+1)}{h}\\&=\frac{2(x^2+2xh+h^2)-5(x+h)+1-(2x^2-5x+1)}{h}\\&=\frac{2x^2+4xh+2h^2-5x-5h+1-2x^2+5x-1}{h}\\&=\frac{4xh+2h^2-5h}{h}\\&=\frac{h(4x+2h-5)}{h}\\&=4x+2h-5\end{aligned}$$

66. $f(x)=3x^2-5x+2$

$$\begin{aligned}&\frac{f(x+h)-f(x)}{h}\\&=\frac{3(x+h)^2-5(x+h)+2-(3x^2-5x+2)}{h}\\&=\frac{3(x^2+2xh+h^2)-5(x+h)+2-(3x^2-5x+2)}{h}\\&=\frac{3x^2+6xh+3h^2-5x-5h+2-3x^2+5x-2}{h}\\&=\frac{6xh+3h^2-5h}{h}\\&=\frac{h(6x+3h-5)}{h}\\&=6x+3h-5\end{aligned}$$

67. $f(x)=7x^2-3x+2$

$$\begin{aligned}\frac{f(x+h)-f(x)}{h}&=\frac{7(x+h)^2-3(x+h)+2-(7x^2-3x+2)}{h}\\&=\frac{7(x^2+2xh+h^2)-3(x+h)+2-(7x^2-3x+2)}{h}\\&=\frac{7x^2+14xh+7h^2-3x-3h+2-7x^2+3x-2}{h}\\&=\frac{14xh+7h^2-3h}{h}\\&=\frac{h(14x+7h-3)}{h}\\&=14x+7h-3\end{aligned}$$

68. $f(x)=4x^2-5x+3$

$$\begin{aligned}&\frac{f(x+h)-f(x)}{h}\\&=\frac{4(x+h)^2-5(x+h)+3-(4x^2-5x+3)}{h}\\&=\frac{4(x^2+2xh+h^2)-5(x+h)+3-(4x^2-5x+3)}{h}\\&=\frac{4x^2+8xh+4h^2-5x-5h+3-4x^2+5x-3}{h}\\&=\frac{8xh+4h^2-5h}{h}\\&=\frac{h(8x+4h-5)}{h}\\&=8x+4h-5\end{aligned}$$

69. $f(x)=x^3$

$$\begin{aligned}\frac{f(x+h)-f(x)}{h}&=\frac{(x+h)^3-x^3}{h}\\&=\frac{x^3+3x^2h+3xh^2+h^3-x^3}{h}\\&=\frac{3x^2h+3xh^2+h^3}{h}\\&=\frac{h(3x^2+3xh+h^2)}{h}\\&=3x^2+3xh+h^2\end{aligned}$$

70. $f(x)=x^4$

$$\begin{aligned}\frac{f(x+h)-f(x)}{h}&=\frac{(x+h)^4-x^4}{h}\\&=\frac{x^4+4x^3h+6x^2h^2+4xh^3+h^4-x^4}{h}\\&=\frac{4x^3h+6x^2h^2+4xh^3+h^4}{h}\\&=\frac{h(4x^3+6x^2h+4xh^2+h^3)}{h}\\&=4x^3+6x^2h+4xh^2+h^3\end{aligned}$$

71. $f(x)=\frac{2}{x}$

$$\begin{aligned}\frac{f(x+h)-f(x)}{h}&=\frac{\frac{2}{x+h}-\frac{2}{x}}{h}\\&=\frac{\frac{2}{x+h}-\frac{2}{x}}{h}\cdot\frac{x(x+h)}{x(x+h)}\\&=\frac{2x-2(x+h)}{hx(x+h)}\\&=\frac{2x-2x-2h}{hx(x+h)}\\&=\frac{-2h}{hx(x+h)}\\&=\frac{-2}{x(x+h)}\end{aligned}$$

72. $f(x)=\frac{3}{x}$

$$\begin{aligned}\frac{f(x+h)-f(x)}{h}&=\frac{\frac{3}{x+h}-\frac{3}{x}}{h}\\&=\frac{\frac{3}{x+h}-\frac{3}{x}}{h}\cdot\frac{x(x+h)}{x(x+h)}\\&=\frac{3x-3(x+h)}{hx(x+h)}\\&=\frac{3x-3x-3h}{hx(x+h)}\\&=\frac{-3h}{hx(x+h)}\\&=\frac{-3}{x(x+h)}\end{aligned}$$

73. $f(x)=\frac{1}{x^2}$

$$\begin{aligned}\frac{f(x+h)-f(x)}{h}&=\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}\\&=\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}\cdot\frac{x^2(x+h)^2}{x^2(x+h)^2}\\&=\frac{x^2-(x+h)^2}{hx^2(x+h)^2}\\&=\frac{x^2-x^2-2xh-h^2}{hx^2(x+h)^2}\\&=\frac{-2xh-h^2}{hx^2(x+h)^2}\\&=\frac{h(-2x-h)}{hx^2(x+h)^2}\\&=\frac{-2x-h}{x^2(x+h)^2}\text{ or }\frac{-2x-h}{x^2(x^2+2xh+h^2)}\\&\text{or }\frac{-2x-h}{x^4+2x^3h+x^2h^2}\end{aligned}$$

74. $f(x)=\sqrt{x}$

$$\frac{f(x+h)-f(x)}{h}=\frac{\sqrt{x+h}-\sqrt{x}}{h}$$

75. **a.** 2.70481
b. 2.71815
c. 2.71828
d. Yes, 2.71828

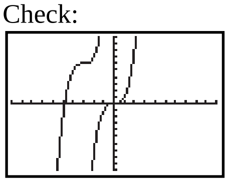
76.

x	$\left(1+\frac{1}{x}\right)^x$
100	2.70481
10,000	2.71815
1,000,000	2.71828
10,000,000	2.71828

Yes, 2.71828

77. The graph of $y=(x+3)^3+6$ is the same shape as the graph of $y=x^3$ but it is shifted left 3 units and up 6 units.

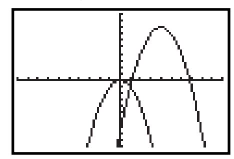
Check:



on [–10, 10] by [–10, 10]

78. The graph of $y=-(x-4)^2+8$ is the same shape as the graph of $y=-x^2$ but it is shifted right 4 units and up 8 units.

Check:



on [–10, 10] by [–10, 10]

79. $P(x)=522(1.0053)^x$

$P(50)=522(1.0053)^{50}\approx 680$ million people in 1750

80. $P(100)=522(1.0053)^{100}$

≈ 886 million people in 1800

81. **a.** For x = 3000, use $f(x) = 0.10x$.

$$f(x)=0.10x$$
$$f(3000)=0.10(3000)=\$300$$

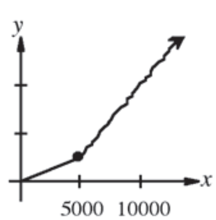
b. For x = 5000, use $f(x) = 0.10x$.

$$f(x)=0.10x$$
$$f(5000)=0.10(5000)=\$500$$

c. For x = 10,000, use $f(x) = 500 + 0.30(x - 5000)$.

$$f(x)=500+0.30(x-5000)$$
$$f(10,000)=500+0.30(10,000-5000)$$
$$=500+0.30(5000)$$
$$=\$2000$$

d.



82. **a.** For x = 3000, use $f(x) = 0.15x$.

$$f(x)=0.15x$$
$$f(3000)=0.15(3000)=\$450$$

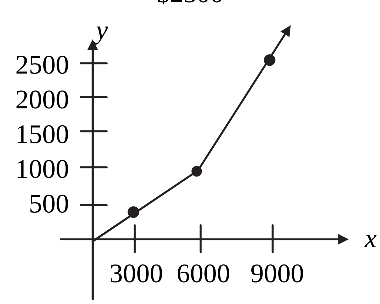
b. For x = 6000, use $f(x) = 0.15x$.

$$f(x)=0.15x$$
$$f(6000)=0.15(6000)=\$900$$

c. For x = 10,000, use $f(x) = 900 + 0.40(x - 6000)$.

$$f(x)=900+0.40(x-6000)$$
$$f(10,000)=900+0.40(10,000-6000)$$
$$=900+0.40(4000)$$
$$=\$2500$$

d.



83. **a.** For $x=\frac{2}{3}$, use $f(x)=10.5x$.

$$\begin{aligned} f(x)&=10.5x \\ f\left(\frac{2}{3}\right)&=10.5\left(\frac{2}{3}\right) \\ &=7 \text{ years} \end{aligned}$$

For $x=\frac{4}{3}$, use $f(x)=10.5x$.

$$\begin{aligned} f(x)&=10.5x \\ f\left(\frac{4}{3}\right)&=10.5\left(\frac{4}{3}\right) \\ &=14 \text{ years} \end{aligned}$$

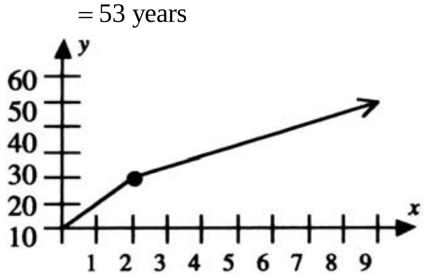
For $x=4$, use $f(x)=21+4(x-2)$.

$$\begin{aligned} f(x)&=21+4(x-2) \\ f(4)&=21+4(4-2) \\ &=21+4(2) \\ &=29 \text{ years} \end{aligned}$$

For $x=10$, use $f(x)=21+4(x-2)$.

$$\begin{aligned} f(x)&=21+4(x-2) \\ f(10)&=21+4(10-2) \\ &=21+4(8) \\ &=53 \text{ years} \end{aligned}$$

b.



84. **a.** For $x=\frac{2}{3}$, use $f(x)=15x$.

$$\begin{aligned} f(x)&=15x \\ f\left(\frac{2}{3}\right)&=15\left(\frac{2}{3}\right) \\ &=10 \text{ years} \end{aligned}$$

For $x=\frac{4}{3}$, use $f(x)=15+9(x-1)$.

$$\begin{aligned} f(x)&=15+9(x-1) \\ f\left(\frac{4}{3}\right)&=15+9\left(\frac{4}{3}-1\right) \\ &=18 \text{ years} \end{aligned}$$

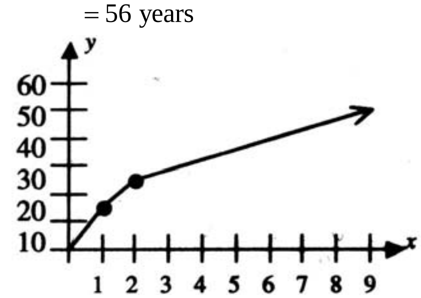
For $x=4$, use $f(x)=15+9(x-1)$.

$$\begin{aligned} f(x)&=15+9(x-1) \\ f(4)&=15+9(4-1) \\ &=15+9(3) \\ &=32 \text{ years} \end{aligned}$$

For $x=10$, use $f(x)=15+9(x-1)$.

$$\begin{aligned} f(x)&=15+9(x-1) \\ f(10)&=15+9(10-1) \\ &=15+9(8) \\ &=56 \text{ years} \end{aligned}$$

b.



85. Substitute $K=24\cdot L^{-1}$ into $3L+8K=48$.

$$\begin{aligned} 3L+8\left(24\cdot L^{-1}\right)&=48 \\ 3L+\frac{192}{L}&=48 \\ 3L^2+192&=48L \\ 3L^2-48L+192&=0 \\ 3\left(L^2-16L+64\right)&=0 \\ 3(L-8)^2&=0 \end{aligned}$$

So, $L=8$.

And $K=24\cdot 8^{-1}=\frac{24}{8}=3$

The intersection point is (8, 3).

86. Substitute $K=180\cdot L^{-1}$ into $5L+4K=120$.

$$\begin{aligned} 5L+4\left(180\cdot L^{-1}\right)&=120 \\ 5L+\frac{720}{L}&=120 \\ 5L^2+720&=120L \\ 5L^2-120L+720&=0 \\ 5\left(L^2-24L+144\right)&=0 \\ 5(L-12)^2&=0 \end{aligned}$$

So, $L=12$.

And $K=180\cdot 12^{-1}=\frac{180}{12}=15$

The intersection point is (12, 15).

87. First find the composition $R(v(t))$.

$$\begin{aligned} R(v(t))&=2\left[v(t)\right]^{0.3} \\ &=2(60+3t)^{0.3} \end{aligned}$$

Then find $R(v(10))$.

$$\begin{aligned} R(v(10))&=2\left(60+3(10)\right)^{0.3}=2(90)^{0.3} \\ &\approx \$7.714 \text{ million} \end{aligned}$$

88. We must find the composition $R(p(t))$.

$$\begin{aligned} R(p(t))&=3\left[p(t)\right]^{0.25} \\ &=3(55+4t)^{0.25} \end{aligned}$$

$$\begin{aligned} R(5)&=3[55+4(5)]^{0.25} \\ &=3(75)^{0.25}\approx \$8.8 \text{ million} \end{aligned}$$

89. **a.** $f(x) = 4^{10} = 1{,}048{,}576$ cells
≈ 1 million cells

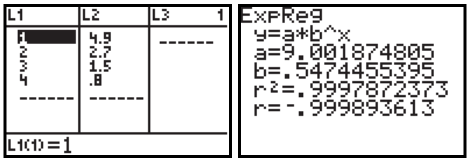
b. $f(15) = 4^{15} = 1{,}073{,}741{,}824$ cells
No, the mouse will not survive beyond day 15.

90. The value 2020 – 2012 = 8 corresponds to the year 2020. Substitute 8 for x.

$f(8) = 226(1.11)^8 \approx \$521$ billion

There will be about \$521 billion of e-commerce in the year 2020.

91. **a.**



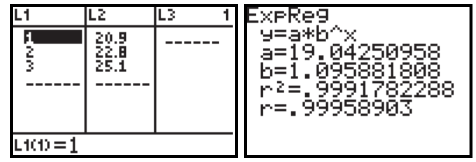
$y = 9 \cdot 0.547^x$ (rounded)

b. For year 2020, $x = 6$.

$y = 9 \cdot 0.547^6 \approx 0.24$ weeks

or less than 2 days

92. **a.**



$y = 19.04 \cdot 1.096^x$ (rounded)

b. For year 2020, $x = 5$.

$y = 19.04 \cdot 1.096^5 \approx 30.1$ million

93. One will have “missing points” at the excluded x-values.

94. $x^{\sqrt{2}} + 1$ is not a polynomial because the exponent is not a non-negative integer.

95. A slope of 1 is a tax of 100%. That means, all dollars taxed are paid as the tax.

96.
$$\begin{aligned} f(f(x)) &= a(f(x)) \\ &= a(ax) \\ &= a^2 x \end{aligned}$$

97.
$$\begin{aligned} f(f(x)) &= (f(x)) + a \\ &= (x+a) + a \\ &= x + 2a \end{aligned}$$

98. $f(x) + 10$ is translated up by 10 units.

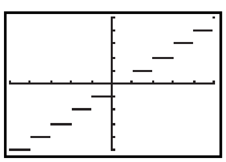
99. $f(x+10)$ is shifted to the left by 10 units.

100. $f(x+10)+10$ is shifted up 10 units and left 10 units.

101. False: $f(x+h) = (x+h)^2 = x^2 + 2xh + h^2$, not $x^2 + h^2$.

102. True:
$$\begin{aligned} f(x+h) &= m(x+h) + b \\ &= mx + b + mh \\ &= f(x) + mh \end{aligned}$$

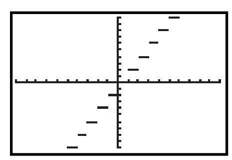
103. **a.**



on [–5, 5] by [–5, 5]
Note that each line segment in this graph includes its left end-point, but excludes its right endpoint. So it should be drawn like ●—○.

b. Domain = $\mathbb{R}$; range = the set of integers

104. **a.**



on [–10, 10] by [–10, 10]

b. Domain = $\mathbb{R}$; range = the set of even integers.

105. **a.**
$$\begin{aligned} f(g(x)) &= a[g(x)] + b = a(cx+d) + b \\ &= acx + ad + b \end{aligned}$$

b. Yes

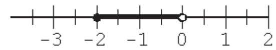
106. **a.** $f(g(x)) = [g(x)]^2 = (x^2)^2 = x^4$

b. Yes, because the composition of two polynomials involves raising integral powers to integral powers.

REVIEW EXERCISES AND CHAPTER TEST FOR CHAPTER 1

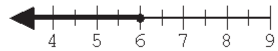
1. $\{x \mid 2 < x \le 5\}$

2. $\{x \mid -2 \le x < 0\}$



3. $\{x \mid x \ge 100\}$

4. $\{x \mid x \le 6\}$



5. Hurricane: $[74, \infty)$; storm: $[55, 74)$; gale: $[38, 55)$; small craft warning: $[21,38)$

6. **a.** $(0, \infty)$
b. $(-\infty, 0)$
c. $[0, \infty)$
d. $(-\infty, 0]$

7.
$$\begin{aligned} y-(-3) &= 2(x-1) \\ y+3 &= 2x-2 \\ y &= 2x-5 \end{aligned}$$

8.
$$\begin{aligned} y-6 &= -3[x-(-1)] \\ y-6 &= -3x-3 \\ y &= -3x+3 \end{aligned}$$

9. Since the vertical line passes through the point with x-coordinate 2, the equation of the line is $x = 2$.

10. Since the horizontal line passes through the point with y-coordinate 3, the equation of the line is
$y = 3$.

11. First, calculate the slope from the two points.
$$m = \frac{-3-3}{2-(-1)} = \frac{-6}{3} = -2$$
Now use the point-slope formula with this slope and the point (–1, 3).
$$\begin{aligned} y-3 &= -2[x-(-1)] \\ y-3 &= -2x-2 \\ y &= -2x+1 \end{aligned}$$

12. First find the slope of the line $x + 2y = 8$.
Write the equation in slope-intercept form.
$$y = -\frac{1}{2}x + 4.$$
The slope of the perpendicular line is $m = 2$.
Next, use the point-slope form with the point (6, –1):
$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y+1 &= 2(x-6) \\ y &= 2x-13 \end{aligned}$$

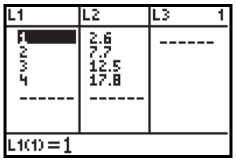
13. Since the y-intercept is (0, –1), $b = -1$. To find the slope, use the slope formula with the points (0, –1) and (1, 1).
$$m = \frac{1-(-1)}{1-0} = 2$$
Thus the equation of the line is $y = 2x - 1$.

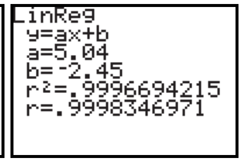
14. Since the y-intercept is (0, 1), $b = 1$. To find the slope, use the slope formula with the points (0, 1) and (2, 0).
$$m = \frac{0-1}{2-0} = -\frac{1}{2}$$
The equation of the line is $y = -\frac{1}{2}x + 1$

15. **a.** Use the straight-line depreciation formula with price = 25,000, useful lifetime = 8, and scrap value = 1000.
$$\begin{aligned} \text{Value} &= \text{price} - \left(\frac{\text{price} - \text{scrap value}}{\text{useful lifetime}}\right)t \\ &= 25{,}000 - \left(\frac{25{,}000-1000}{8}\right)t \\ &= 25{,}000 - \left(\frac{24{,}000}{8}\right)t \\ &= 25{,}000 - 3000t \end{aligned}$$
b.
$$\begin{aligned} \text{Value after 4 years} &= 25{,}000 - 3000(4) \\ &= 25{,}000 - 12{,}000 \\ &= \$13{,}000 \end{aligned}$$

16. **a.** Use the straight-line depreciation formula with price = 78,000, useful lifetime = 15, and scrap value = 3000.
$$\begin{aligned} \text{Value} &= \text{price} - \left(\frac{\text{price} - \text{scrap value}}{\text{useful lifetime}}\right)t \\ &= 78{,}000 - \left(\frac{78{,}000-3000}{15}\right)t \\ &= 78{,}000 - \left(\frac{75{,}000}{15}\right)t \\ &= 78{,}000 - 5000t \end{aligned}$$
b.
$$\begin{aligned} \text{Value after 8 years} &= 78{,}000 - 5000(8) \\ &= 78{,}000 - 40{,}000 \\ &= \$38{,}000 \end{aligned}$$

17. **a.**





$y = 5.04x - 2.45$

b. The number is increasing by about 5000 3D screens per year.

c. For the year 2020, $x = 12$
$y = 5.04(12) - 2.45 = 58.03$
or about 58 thousand

18. $\left(\frac{6}{1}\right)^2 = \frac{6^2}{1^2} = 36$

19. $\left(\frac{4}{3}\right)^{-1} = \frac{3}{4}$

20. $64^{1/2} = \sqrt{64} = 8$

21. $1000^{1/3} = \sqrt[3]{1000} = 10$

22. $(81)^{-3/4} = \left(\frac{1}{81}\right)^{3/4} = \left(\sqrt[4]{\frac{1}{81}}\right)^3 = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$

23. $100^{-3/2} = \left(\frac{1}{100}\right)^{3/2} = \left(\sqrt{\frac{1}{100}}\right)^3 = \left(\frac{1}{10}\right)^3 = \frac{1}{1000}$

24. $\left(-\frac{8}{27}\right)^{-2/3} = \left(-\frac{27}{8}\right)^{2/3} = \left(\sqrt[3]{-\frac{27}{8}}\right)^2$
$= \left(-\frac{3}{2}\right)^2 = \frac{9}{4}$

25. $\left(\frac{9}{16}\right)^{-3/2} = \left(\frac{16}{9}\right)^{3/2} = \left(\sqrt{\frac{16}{9}}\right)^3 = \left(\frac{4}{3}\right)^3 = \frac{64}{27}$

26. 13.97

27. 112.32

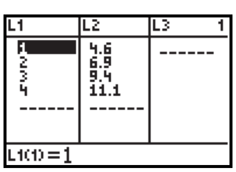
28. **a.** $y = 0.86x^{0.47}$
$y = 0.86(4000)^{0.47} = 42.4$
The weight for the top cold-blooded meat-eating animals in Hawaii is 42.4 lbs.

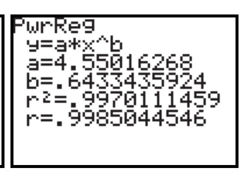
b. $y = 0.86x^{0.47}$
$y = 0.86(9{,}400{,}000)^{0.47} = 1628.8$
The weight for the top cold-blooded meat-eating animals in North America is 1628.8 lbs.

29. **a.** $y = 1.7x^{0.52}$
$y = 1.7(4000)^{0.52} = 126.9$
The weight for the top warm-blooded plant-eating animals in Hawaii is 126.9 lbs.

b. $y = 1.7x^{0.52}$
$y = 1.7(9{,}400{,}000)^{0.52} = 7185.8$
The weight for the top warm-blooded plant-eating animals in Hawaii is 7185.8 lbs.

30. **a.**





$y = 4.55x^{0.643}$ (rounded)

b. For year 2020, $x = 12$.
$y = 4.55(12)^{0.643} \approx \$22.5$ billion

31. **a.** $f(11) = \sqrt{11-7} = \sqrt{4} = 2$

b. Domain = $\{x \mid x \geq 7\}$ because $\sqrt{x-7}$ is defined only for all values of $x \geq 7$.

c. Range = $\{y \mid y \geq 0\}$

32. **a.** $g(-1) = \frac{1}{-1+3} = \frac{1}{2}$

b. Domain = $\{t \mid t \neq -3\}$

c. Range = $\{y \mid y \neq 0\}$

33. **a.** $h(16) = 16^{-3/4} = \left(\frac{1}{16}\right)^{3/4} = \left(\sqrt[4]{\frac{1}{16}}\right)^3$

$= \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

b. Domain = $\{w \mid w > 0\}$ because the fourth root is defined only for nonnegative numbers and division by 0 is not defined.

c. Range = $\{y \mid y > 0\}$

34. **a.** $w(8) = 8^{-4/3} = \left(\frac{1}{8}\right)^{4/3} = \left(\sqrt[3]{\frac{1}{8}}\right)^4$

$= \left(\frac{1}{2}\right)^4 = \frac{1}{16}$

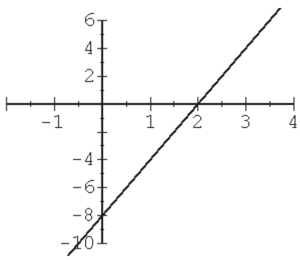
b. Domain = $\{z \mid z \neq 0\}$ because division by 0 is not defined.

c. Range = $\{y \mid y > 0\}$

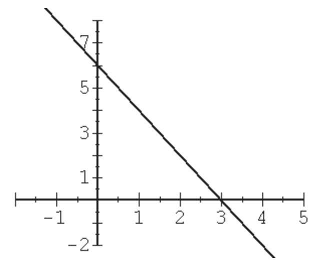
35. Yes

36. No

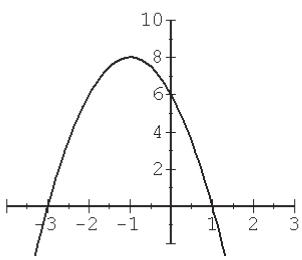
37.



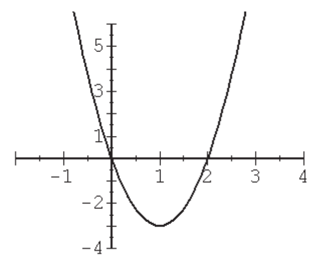
38.



39.



40.



41. **a.**

$3x^2 + 9x = 0$

$3x(x+3) = 0$

Equals 0 at $x = 0$; Equals 0 at $x = -3$

$x = 0$ and $x = -3$

b. Use the quadratic formula with $a = 3$, $b = 9$, and $c = 0$

$$\frac{-9 \pm \sqrt{9^2 - 4(3)(0)}}{2(3)} = \frac{-9 \pm \sqrt{81}}{6} = \frac{-9 \pm 9}{6} = 0, -3$$

$x = 0$ and $x = -3$

42. **a.**

$2x^2 - 8x - 10 = 0$

$2(x^2 - 4x - 5) = 0$

$(x-5)(x+1) = 0$

Equals 0 at $x = 5$; Equals 0 at $x = -1$

$x = 5$ and $x = -1$

b. Use the quadratic formula with $a = 2$, $b = -8$, and $c = -10$

$$\frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(-10)}}{2(2)} = \frac{8 \pm \sqrt{64+80}}{4} = \frac{8 \pm 12}{4} = 5, -1$$

$x = 5$ and $x = -1$

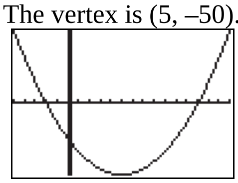
43. **a.**
$$3x^2+3x+5=11$$
$$3x^2+3x-6=0$$
$$3(x^2+x-2)=0$$
$$(x+2)(x-1)=0$$
Equals 0 at $x=-2$ Equals 0 at $x=1$
$x=-2$ and $x=1$

b.
$$\frac{-3\pm\sqrt{3^2-4(3)(-6)}}{2(3)}=\frac{-3\pm\sqrt{9+72}}{6}=\frac{-3\pm\sqrt{81}}{6}=\frac{-3\pm 9}{6}=-2,1$$
$x=-2$ and $x=1$

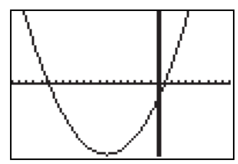
44. **a.**
$$4x^2-2=2$$
$$4x^2=4$$
$$x^2=1$$
$$x=\pm 1$$
$x = 1$ and $x = -1$

b.
$$\frac{-0\pm\sqrt{0^2-4(4)(-4)}}{2(4)}=\frac{\pm\sqrt{64}}{8}=\frac{\pm 8}{8}=\pm 1$$
$x = 1$ and $x = -1$

45. **a.** Use the vertex formula with $a = 1$ and $b = -10$.
$$x=\frac{-b}{2a}=\frac{-(-10)}{2(1)}=\frac{10}{2}=5$$
To find y, evaluate $f(5)$.
$$f(5)=(5)^2-10(5)-25=-50$$
The vertex is $(5, -50)$.

b.

on $[-5, 15]$ by $[-50, 50]$

46. **a.** Use the vertex formula with $a = 1$ and $b = 14$.
$$x=\frac{-b}{2a}=\frac{-14}{2(1)}=-7$$
To find y, evaluate $f(-7)$.
$$f(-7)=(-7)^2+14(-7)-15=-64$$
The vertex is $(-7, -64)$

b.

on $[-20, 10]$ by $[-65, 65]$

47. Let x = number of miles per day.
$C(x) = 0.12x + 45$

48. Use the interest formula with $P = 10{,}000$ and $r = 0.08$.
$I(t) = 10{,}000(0.08)t = 800t$

49. Let x = the altitude in feet.
$$T(x)=70-\frac{x}{300}$$

50. Let t = the number of years after 2010.
$$C(t)=0.45t+20.3$$
$$25=0.45t+20.3$$
$t\approx 10.4$ years after 2010; in the year 2020

51. **a.** To find the break even points, solve the equation $C(x) = R(x)$ for x.
$$C(x)=R(x)$$
$$80x+1950=-2x^2+240x$$
$$2x^2-160x+1950=0$$
$$x^2-80x+975=0$$
$$(x-65)(x-15)=0$$
Equals 0 at $x=65$ Equals 0 at $x=15$
$x=65$ and $x=15$
The store breaks even at 15 receivers and at 65 receivers.

52. **a.** To find the break even points, solve the equation $C(x) = R(x)$ for x.
$$C(x)=R(x)$$
$$220x+202{,}500=-3x^2+2020x$$
$$3x^2-1800x+202{,}500=0$$
$$x^2-600x+67{,}500=0$$
$$(x-450)(x-150)=0$$
Equals 0 at $x=450$ Equals 0 at $x=150$
$x=450$ and $x=150$
The outlet breaks even at 150 units and 450 units.

51. **b.** To find the number of receivers that maximizes profit, first find the profit function, $P(x) = R(x) - C(x)$.

$$\begin{aligned} P(x) &= \left(-2x^2 + 240x\right) - (80x + 1950) \\ &= -2x^2 + 160x - 1950 \end{aligned}$$

Since this is a parabola that opens downward, the maximum profit is found at the vertex.

$$x = \frac{-b}{2a} = \frac{-160}{2(-2)} = \frac{-160}{-4} = 40$$

Thus, profit is maximized when 40 receivers are installed per week. The maximum profit is found by evaluating $P(40)$.

$$\begin{aligned} P(40) &= -2(40)^2 + 160(40) - 1950 \\ &= \$1250 \end{aligned}$$

Therefore, the maximum profit is $1250.

52. **b.** To find the number of units that maximizes profit, first find the profit function, $P(x) = R(x) - C(x)$.

$$\begin{aligned} P(x) &= \left(-3x^2 + 2020x\right) - (220x + 202,500) \\ &= -3x^2 + 1800x - 202,500 \end{aligned}$$

Since this is a parabola that opens downward, the maximum profit is found at the vertex.

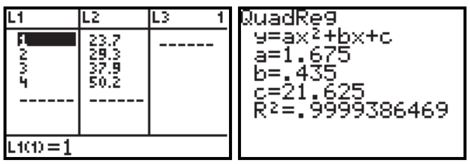
$$x = \frac{-b}{2a} = \frac{-1800}{2(-3)} = \frac{-1800}{-6} = 300$$

Thus, profit is maximized when 300 units are installed per month. The maximum profit is found by evaluating $P(300)$.

$$\begin{aligned} P(300) &= -3(300)^2 + 1800(300) - 202,500 \\ &= \$67,500 \end{aligned}$$

Therefore, the maximum profit is $67,500.

53. **a.**



$y = 1.675x^2 + 0.435x + 21.625$ (rounded)

b. For year 2020, $x = 12$.

$$\begin{aligned} y &= 1.675(12)^2 + 0.435(12) + 21.6 \\ &\approx \$268 \text{ billion} \end{aligned}$$

54. **a.** $f(-1) = \dfrac{3}{(-1)(-1-2)} = \dfrac{3}{3} = 1$

b. Domain = $\{x \mid x \neq 0, x \neq 2\}$

c. Range = $\{y \mid y > 0 \text{ or } y \leq -3\}$

55. **a.** $f(-8) = \dfrac{16}{(-8)(-8+4)} = \dfrac{16}{32} = \dfrac{1}{2}$

b. Domain = $\{x \mid x \neq 0, x \neq -4\}$

c. Range = $\{y \mid y > 0 \text{ or } y \leq -4\}$

56. **a.** $g(-4) = |-4+2| - 2 = 2 - 2 = 0$

b. Domain = $\Re$

c. Range = $\{y \mid y \geq 0\}$

57. **a.** $g(-5) = (-5) - |-5| = -5 - 5 = -10$

b. Domain = $\Re$

c. Range = $\{y \mid y \geq 0\}$

58.

$$\begin{aligned} 5x^4 + 10x^3 &= 15x^2 \\ 5x^4 + 10x^3 - 15x^2 &= 0 \\ 5x^2\left(x^2 + 2x - 3\right) &= 0 \\ x^2(x+3)(x-1) &= 0 \end{aligned}$$

Equations 0 at $x = 0$ Equals 0 at $x = -3$ Equals 0 at $x = 1$

$x = 0$, $x = -3$, and $x = 1$

59.

$$\begin{aligned} 4x^5 + 8x^4 &= 32x^3 \\ 4x^5 + 8x^4 - 32x^3 &= 0 \\ 4x^3\left(x^2 + 2x - 8\right) &= 0 \\ x^3(x+4)(x-2) &= 0 \end{aligned}$$

Equals 0 at $x = 0$ Equals 0 at $x = -4$ Equals 0 at $x = 2$

$x = 0$, $x = -4$, and $x = 2$

60.

$$\begin{aligned} 2x^{5/2} - 8x^{3/2} &= 10x^{1/2} \\ 2x^{5/2} - 8x^{3/2} - 10x^{1/2} &= 0 \\ 2x^{1/2}(x^2 - 4x - 5) &= 0 \\ x^{1/2}(x-5)(x+1) &= 0 \end{aligned}$$

Equals 0 at $x = 0$ Equals 0 at $x = 5$ Equals 0 at $\cancel{x = -1}$

$x = 0$, $x = 5$ and $\cancel{x = -1}$

Only $x = 0$ and $x = 5$ are solutions.

61.

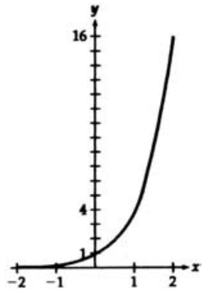
$$\begin{aligned} 3x^{5/2} + 3x^{3/2} &= 18x^{1/2} \\ 3x^{5/2} + 3x^{3/2} - 18x^{1/2} &= 0 \\ 3x^{1/2}(x^2 + x - 6) &= 0 \\ x^{1/2}(x+3)(x-2) &= 0 \end{aligned}$$

Equals 0 at $x = 0$ Equals 0 at $\cancel{x = -3}$ Equals 0 at $x = 2$

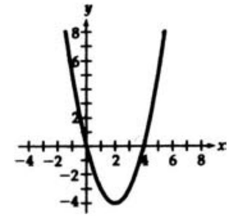
$x = 0$, $\cancel{x = -3}$ and $x = 2$

Only $x = 0$ and $x = 2$ are solutions.

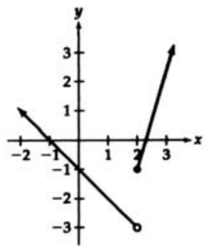
62.



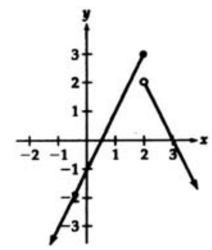
63.



64.



65.



66. **a.**
$$f(g(x)) = [g(x)]^2 + 1 = \left(\frac{1}{x}\right)^2 + 1 = \frac{1}{x^2} + 1$$

b. $$g(f(x)) = \frac{1}{f(x)} = \frac{1}{x^2+1}$$

67. **a.** $$f(g(x)) = \sqrt{g(x)} = \sqrt{5x-4}$$

b. $$g(f(x)) = 5[f(x)] - 4 = 5\sqrt{x} - 4$$

68. **a.** $$f(g(x)) = \frac{g(x)+1}{g(x)-1} = \frac{x^3+1}{x^3-1}$$

b. $$g(f(x)) = [f(x)]^3 = \left(\frac{x+1}{x-1}\right)^3$$

69. **a.** $$f(g(x)) = |g(x)| = |x+2|$$

b. $$g(f(x)) = [f(x)] + 2 = |x| + 2$$

70.
$$f(x) = 2x^2 - 3x + 1$$
$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{2(x+h)^2 - 3(x+h) + 1 - (2x^2 - 3x + 1)}{h}\\ &= \frac{2(x^2 + 2xh + h^2) - 3(x+h) + 1 - (2x^2 - 3x + 1)}{h}\\ &= \frac{2x^2 + 4xh + 2h^2 - 3x - 3h + 1 - 2x^2 + 3x - 1}{h}\\ &= \frac{4xh + 2h^2 - 3h}{h}\\ &= \frac{h(4x + 2h - 3)}{h}\\ &= 4x + 2h - 3\end{aligned}$$

71.
$$f(x) = \frac{5}{x}$$
$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{\frac{5}{x+h} - \frac{5}{x}}{h}\\ &= \frac{\frac{5}{x+h} - \frac{5}{x}}{h} \cdot \frac{x(x+h)}{x(x+h)}\\ &= \frac{5x - 5(x+h)}{hx(x+h)}\\ &= \frac{5x - 5x - 5h}{hx(x+h)}\\ &= \frac{-5h}{hx(x+h)}\\ &= \frac{-5}{x(x+h)}\end{aligned}$$

72. The advertising budget A as a function of t is the composition of $A(p)$ and $p(t)$.

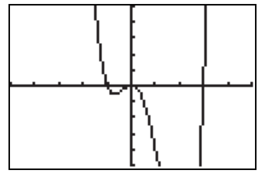
$$A\big(p(t)\big)=2\big[p(t)\big]^{0.15}=2(18+2t)^{0.15}$$
$$A(4)=2\big[18+2(4)\big]^{0.15}=2(26)^{0.15}$$
$$\approx \$3.26 \text{ million}$$

73. **a.**
$$x^4-2x^3-3x^2=0$$
$$x^2(x^2-2x-3)=0$$
$$x^2(x-3)(x+1)=0$$

Equals 0 at $x=0$; Equals 0 at $x=3$; Equals 0 at $x=-1$

$x=0$, $x=3$ and $x=-1$

b.



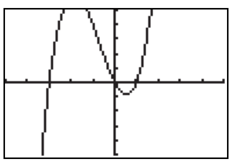
on [–5, 5] by [–5, 5]

74. **a.**
$$x^3+2x^2-3x=0$$
$$x\left(x^2+2x-3\right)=0$$
$$x(x+3)(x-1)=0$$

Equals 0 at $x=0$; Equals 0 at $x=-3$; Equals 0 at $x=1$

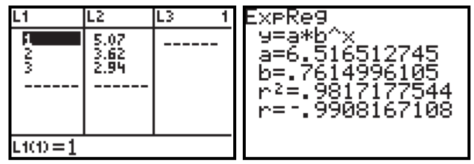
$x=0$, $x=-3$, and $x=1$

b.



on [–5, 5] by [–5, 5]

75. **a.**



$y=6.52\cdot 0.761^x$

b. For year 2020, $x = 4$.

$y=6.52\cdot 0.761^4\approx 2.2$

2.2 crimes per 100,000